Simultaneous control of modes with multiple toroidal periodicity in tokamak plasmas

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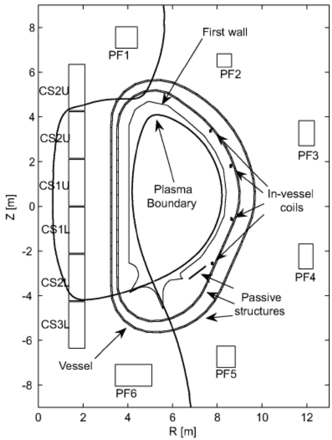
KoM – Fast MPC for Magnetic Plasma Control

Outline

1. Introduction
2. Controller Architecture
3. Plant Model
4. Controller Design
5. Simulation Results

Resistive Wall Modes - 1



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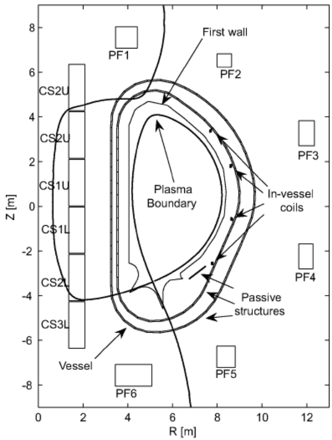
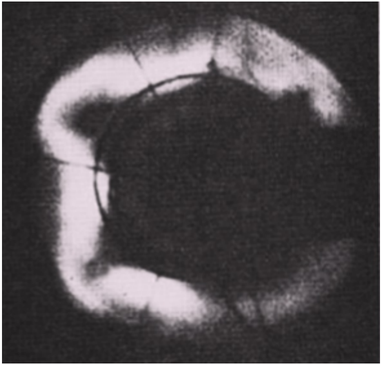


- Tokamak control systems have to deal with different kinds of instabilities related to the presence of a resistive wall that surrounds the plasma
- The main instability is due to an axisymmetric ($n = 0$) mode, the so-called axisymmetric Vertical Displacement Event, which occurs whenever a plasma with a vertical elongated poloidal cross-section is operated

Resistive Wall Modes - 2



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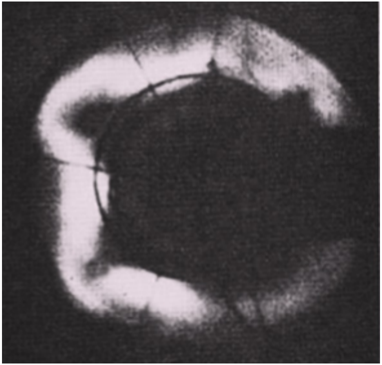

- Another important plasma instability is the one called kink instability, which is the main non-axisymmetric ($n = 1$) mode
- The kink instability arises when the *plasma pressure* exceeds a certain threshold $\rightarrow$ it is similar to a garden hose kinking when it is suddenly pressurized

Control of RWMs

- Elongated plasmas enable to increase the energy confinement time, which is an essential criterion for realizing sustained fusion, but they are vertically unstable
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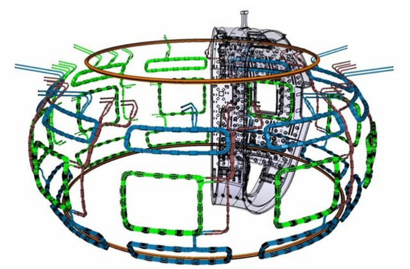
  - The use of an active feedback system, usually called **vertical stabilization system** is required
- Modern tokamak devices operate at high plasma pressure, hence a kink instability is most likely to occur
  - a **control system to stabilize also the** $n = 1$ **mode** becomes necessary

Control of RWMs in ITER

- In this paper we propose a control architecture that enables to control $n = 0$ and $n = 1$ instabilities in ITER
- The proposed solution allows us to minimize the control effort in terms of amplitude of the currents in the coils

Control coils

- The stabilization of the $n = 0$ mode is achieved by using the axisymmetric in-vessel coils, which is referred to as VS3 circuit
- The 27 non-axisymmetric coils so-called ELM coils are used to stabilize the $n = 1$ mode. The ELM coils are three for each of the nine sectors; the sectors are equally-spaced and located at the toroidal angles $\eta_i = 40^o \cdot (i - 1)$, with $i = 1, \ldots, 9$



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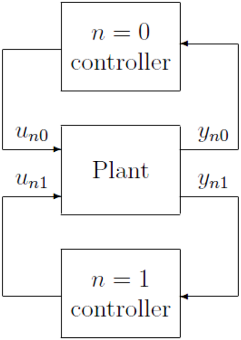
- When designing the controller there are thermal constraints that must be taken into account, and that limit the *rms* value of the current in these internal copper coils
- Two separate controllers have been designed
  - the $n = 0$ controller stabilizes the $n = 0$ mode trying not to induce any $n = 1$ mode and keeping the currents in the ELM coils as low as possible
  - The $n = 1$ controller is an LQ optimal controller which uses the ELM coils. The voltages applied to these coils are distributed in a sinusoidal pattern in the attempt of not inducing any $n = 0$ mode



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$$u_i = \Theta \cdot \begin{pmatrix} u_{Ai} \\ u_{Bi} \end{pmatrix} + h u_{i0}\,, \quad i = 1,2,3\,, \qquad (1)$$

where $h \in \mathbb{R}^{9\times 1}$ is a vector whose elements are all equal to 1, and

$$\Theta = \begin{pmatrix} \cos\eta_1 & \sin\eta_1 \\ \cos\eta_2 & \sin\eta_2 \\ \dots & \dots \\ \cos\eta_9 & \sin\eta_9 \end{pmatrix} \in \mathbb{R}^{9\times 2}\,.$$

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- The u_{i0} terms are used by the $n = 0$ mode stabilization controller in order to minimize the amplitude of the $i_{ELM_{0,i}}$ currents
- The u_{Ai} and u_{Bi} terms counteract the $n = 1$ perturbation without stimulating the $n = 0$ mode

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$$z_i = z_0 + z_A \cos \varphi_i + z_B \sin \varphi_i \,, \tag{2}$$

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- z_A and z_B in (2) are given by

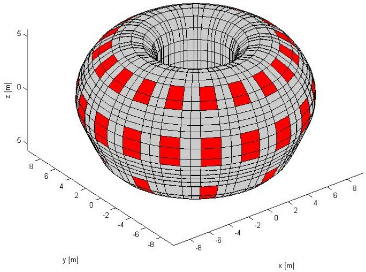
$$\begin{pmatrix} z_A \\ z_B \end{pmatrix} = M^{\dagger} \begin{pmatrix} z_1 - z_0 \\ z_2 - z_0 \\ z_3 - z_0 \end{pmatrix} \,, \tag{3}$$

where

$$M = \begin{pmatrix} \cos\varphi_1 & \sin\varphi_1 \\ \cos\varphi_2 & \sin\varphi_2 \\ \cos\varphi_3 & \sin\varphi_3 \end{pmatrix}$$

Plant model - 1

- The ITER tokamak has been discretized with a 3D finite elements mesh, made of 4970 hexahedral elements, giving rise to $N = 4135$ discrete degrees of freedom
- The mesh takes approximately into account the presence of ports and port extensions, using some conducting patches on the vessel with an equivalent resistivity (shown in red)
- The considered plasma equilibrium is a $I_p = 9\ MA$ configuration, with a normalized $\beta_N = 2.94$ (this parameter quantifies the plasma pressure)



Plant model - 2

For controller design purposes the following linearized model can be considered

$$\dot{x} = Ax + B_{n0}u_{n0} + B_{n1}u_{n1} \tag{4a}$$

$$\begin{pmatrix} y_{n0} \\ y_{n1} \end{pmatrix} = \begin{pmatrix} C_{n0} \\ C_{n1} \end{pmatrix} x + \begin{pmatrix} D_{n0} \\ 0 \end{pmatrix} u_{n0} \tag{4b}$$

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Given the 3D finite elements discretization, the order of the model is about four thousand.

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- Each of the related eigenvectors corresponds to a specific current pattern inside the three-dimensional structure
- The first unstable eigenvalue (around $5.6s^{-1}$) shows an almost axisymmetric current density pattern, and hence corresponds to the $n = 0$ RWM (VDE)
- The other two unstable modes have coinciding values (around $17s^{-1}$) and correspond to two $n = 1$ current density patterns (external kink), which are identical apart from a shift of $\pi/2$ in the toroidal direction

Observability and controllability of the unstable modes

- The two $n = 1$ unstable modes are structurally neither controllable from u_{n0} nor observable from y_{n0}
- Similarly the $n = 0$ unstable mode is neither controllable from u_{n1} nor observable from y_{n1}

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- The two $n = 1$ unstable modes are structurally neither controllable from u_{n0} nor observable from y_{n0}
- Similarly the $n = 0$ unstable mode is neither controllable from u_{n1} nor observable from y_{n1}
- The controller design is split in the design of two separate stabilizing controller, for the $n = 0$ and $n = 1$ modes, respectively

Minimal realizations of the plant model

- The $n = 0$ controller is designed considering the following state space model

$$\dot{\xi} = \widehat{A}_{n0}\xi + \widehat{B}_{n0}u_{n0} \quad (5a)$$

$$y_{n0} = \widehat{C}_{n0}\xi + \widehat{D}_{n0}u_{n0} \quad (5b)$$

where $\widehat{A}_{n0}$, $\widehat{B}_{n0}$, $\widehat{C}_{n0}$, and $\widehat{D}_{n0}$ correspond to a minimal realization of (4) as seen from input u_{n0} to output y_{n0}

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- Similarly, the $n = 1$ controller is designed on the basis of the plant

$$\dot{\zeta} = \widehat{A}_{n1}\zeta + \widehat{B}_{n1}u_{n1} \quad (6a)$$

$$y_{n1} = \widehat{C}_{n1}\zeta \quad (6b)$$

where $\widehat{A}_{n1}$, $\widehat{B}_{n1}$, and $\widehat{C}_{n1}$ are the matrices of a minimal realization of the plant (4) as seen from input u_{n1} to output y_{n1}

The $n = 0$ controller

- Given the high order of model (5), the design of the $n = 0$ controller has been carried out on the basis of the reduced order model

$$\dot{\tilde{\xi}} = \widetilde{A}_{n0}\tilde{\xi} + \widetilde{B}_{n0}u_{n0} \,, \quad \tilde{\xi}(t_0) = \tilde{\xi}_0 \tag{7a}$$

$$y_{n0} = \widetilde{C}_{n0}\tilde{\xi} + \widetilde{D}_{n0}u_{n0} \,, \tag{7b}$$

where $\tilde{\xi} \in \mathbb{R}^{n_r}$ (with n_r about 20)

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- The controller has been designed as a state feedback controller

$$u_{n0} = K_{n0}\tilde{\xi}\,, \tag{8}$$

where the control gain matrix $K_{n0} \in \mathbb{R}^{4\times n_r}$ is chosen such that

$$\sup_{t\in[0,+\infty[} \frac{y_{n0}^T(t)Qy_{n0}(t)}{\tilde{\xi}_0^T R\tilde{\xi}_0} < \gamma_{n0}\,, \tag{9}$$

with $Q \in \mathbb{R}^{5\times 5}$ and $R \in \mathbb{R}^{n_r\times n_r}$ being two positive definite matrices and $\gamma_{n0} > 0$

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- In this way, we try to keep the currents both in the VS3 and in the ELM coils as low as possible

Design theorem

Theorem

Let us consider closed loop system

$$\dot{\tilde{\xi}} = (\widetilde{A}_{n0} + \widetilde{B}_{n0} K_{n0})\tilde{\xi}\,, \quad \tilde{\xi}(t_0) = \tilde{\xi}_0 \tag{10a}$$

$$y_{n0} = (\widetilde{C}_{n0} + \widetilde{D}_{n0} K_{n0})\tilde{\xi} \tag{10b}$$

and condition (9)*. If there exist a positive definite matrix* $Y \in \mathbb{R}^{n_r \times n_r}$ *and a matrix* $W \in \mathbb{R}^{4 \times n_r}$ *such that*

$$\widetilde{A}_{n0} Y + Y \widetilde{A}_{n0}^T + \widetilde{B}_{n0} W + W^T \widetilde{B}_{n0}^T < 0\,, \tag{11a}$$

$$\begin{pmatrix} Q^{-1} & \widetilde{C}_{n0} Y + \widetilde{D}_{n0} W \\ (\widetilde{C}_{n0} Y + \widetilde{D}_{n0} W)^T & Y \end{pmatrix} > 0\,, \tag{11b}$$

$$Y > (\gamma_{n0} R)^{-1}\,, \tag{11c}$$

then system (10) *with* $K_{n0} = WY^{-1}$ *satisfies condition* (9)

$n = 0$ controller - comments

- The approach used is similar to LQR control, the difference being in the fact that with LQR an integral quadratic performance index is minimized, whereas in our design we decided to minimize an L_∞-type norm. This choice is motivated by the fact that the main constraint in the design of the controller are the maximum values reached by the currents flowing in the ELM coils

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- In order to use the state feedback controller (8), an observer of the reduced plant (7) has been designed as a Kalman filter
- Since we are interested in minimizing the output y_{n0} norm in the presence of a VDE, the weighting matrix R in (9) as been chosen as $R = \tilde{\xi}_{VDE}\tilde{\xi}_{VDE}^T + \varepsilon I$, where $\tilde{\xi}_{VDE}$ is the initial state corresponding to a VDE, while the term εI is needed to guarantee the full rank of R

The $n = 1$ controller

- Similarly to what has been done in the $n = 0$ case, the design of the $n = 1$ controller has been carried out considering the reduced order model

$$\dot{\tilde{\zeta}} = \widetilde{A}_{n1}\tilde{\zeta} + \widetilde{B}_{n1}u_{n1} \tag{12a}$$

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- The $n = 1$ controller has been designed as a state feedback controller, where the control matrix K_{n1} has been chosen in order to take into account the saturation of the ELM coil voltages (see **Hu and Lin, IJRNC, 2001**), and to semi-globally stabilize the plant (12) on its null controllable region

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$$\dot{\tilde{\zeta}} = \widetilde{A}_{n1}\tilde{\zeta} + \widetilde{B}_{n1}u_{n1} \quad (12a)$$

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- The $n = 1$ controller has been designed as a state feedback controller, where the control matrix K_{n1} has been chosen in order to take into account the saturation of the ELM coil voltages (see **Hu and Lin, IJRNC, 2001**), and to semi-globally stabilize the plant (12) on its null controllable region
- As for the $n = 0$ controller, a state observer has been designed as a Kalman filter

Controller design

- The linearized model around the equilibrium with $I_p = 9\ MA$ and normalized beta $\beta_N = 2.94$ has been considered
- The order of the model is 4135
- The order of the model has been reduced to 20 for the $n = 0$ controller and to 46 for the $n = 1$ controller

VDE event - 1

- A VDE event is considered consisting of a 10 cm displacement along the unstable $n = 0$ mode with a simple $n = 0$ controller in the form

$$u_{\mathrm{VS3}} = k_1 \dot{z}_0 + k_2 i_{\mathrm{VS3}}$$

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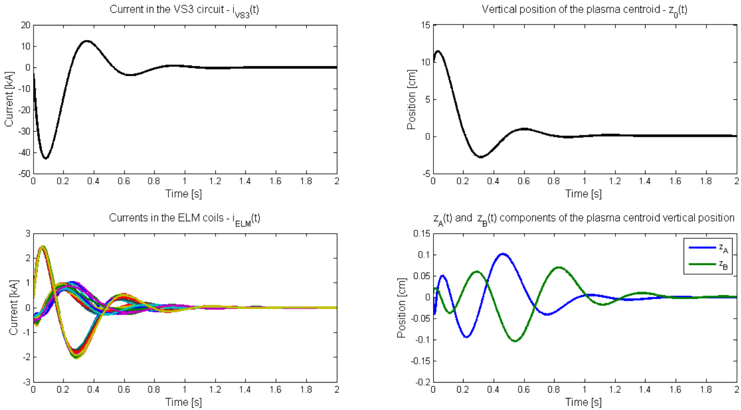
- Scope of this simulation is to show that this $n = 0$ controller gives rise to large currents in the ELM coils
- Indeed in this case the maximum values of the current in the ELM coils is about 2.5 kA

VDE event - 2


Current in the VS3 circuit - $i_{VS3}(t)$
Current [kA]
Time [s]
Vertical position of the plasma centroid - $z_0(t)$
Position [cm]
Time [s]
Currents in the ELM coils - $i_{ELM}(t)$
Current [kA]
Time [s]
$z_A(t)$ and $z_B(t)$ components of the plasma centroid vertical position
Position [cm]
Time [s]
z_A
z_B

VDE event - 3

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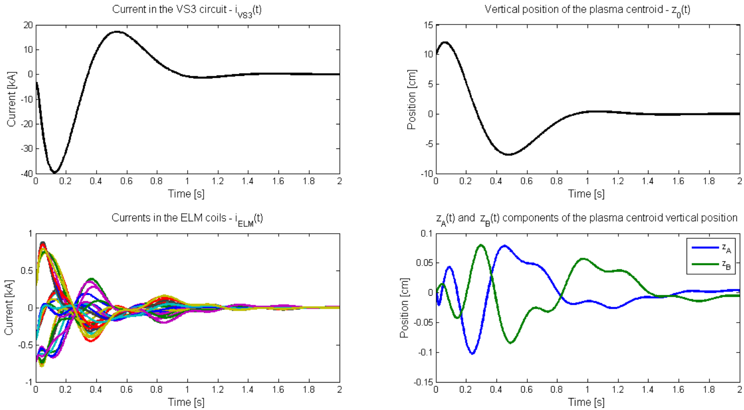
VDE event - 3

- In this simulation the same VDE is considered, but with the controller designed as described before
- Scope of this simulation is to show that resorting to a controller minimizing the index (9), the currents in the ELM coils are significantly reduced with respect to the previous case
- Indeed, in this case the currents in the ELM coils remain well below 1 kA also during the transients

VDE event - 4


Current in the VS3 circuit - $i_{VS3}(t)$
Current [kA]
Time [s]
Vertical position of the plasma centroid - $z_0(t)$
Position [cm]
Time [s]
Currents in the ELM coils - $i_{ELM}(t)$
Current [kA]
Time [s]
$z_A(t)$ and $z_B(t)$ components of the plasma centroid vertical position
z_A
z_B
Position [cm]
Time [s]

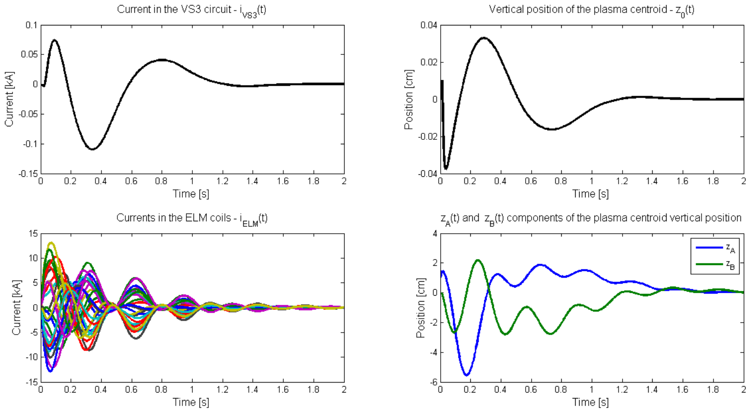
Kink instability - 1

- In the last simulation, a disturbance along the $n = 1$ mode corresponding to a 1 cm displacement at the toroidal angle $\varphi_1 = 0^o$ is applied

Kink instability - 1

- In the last simulation, a disturbance along the $n = 1$ mode corresponding to a 1 cm displacement at the toroidal angle $\varphi_1 = 0^o$ is applied
- It is shown that the proposed architecture produces very little influence of the $n = 1$ loop on the $n = 0$ mode $\rightarrow$ the maximum variation of z_0 is less than one millimeter

Kink instability - 2



Conclusions

- Two separate control loops have been proposed for the simultaneous control of vertical and kink instabilities in ITER
- Scope of the proposed control architecture is to stabilize the plant, maximizing the operating region and minimizing the interaction between the two phenomena
- Simulation results, obtained for a suitable configuration of an ITER plasma, show the effectiveness of the proposed approach