EUROfusion

Fast Model Predictive Control for Magnetic Plasma Control

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SLOVENIAN RESEARCH AGENCY

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Model predictive control of plasma current and shape for ITER

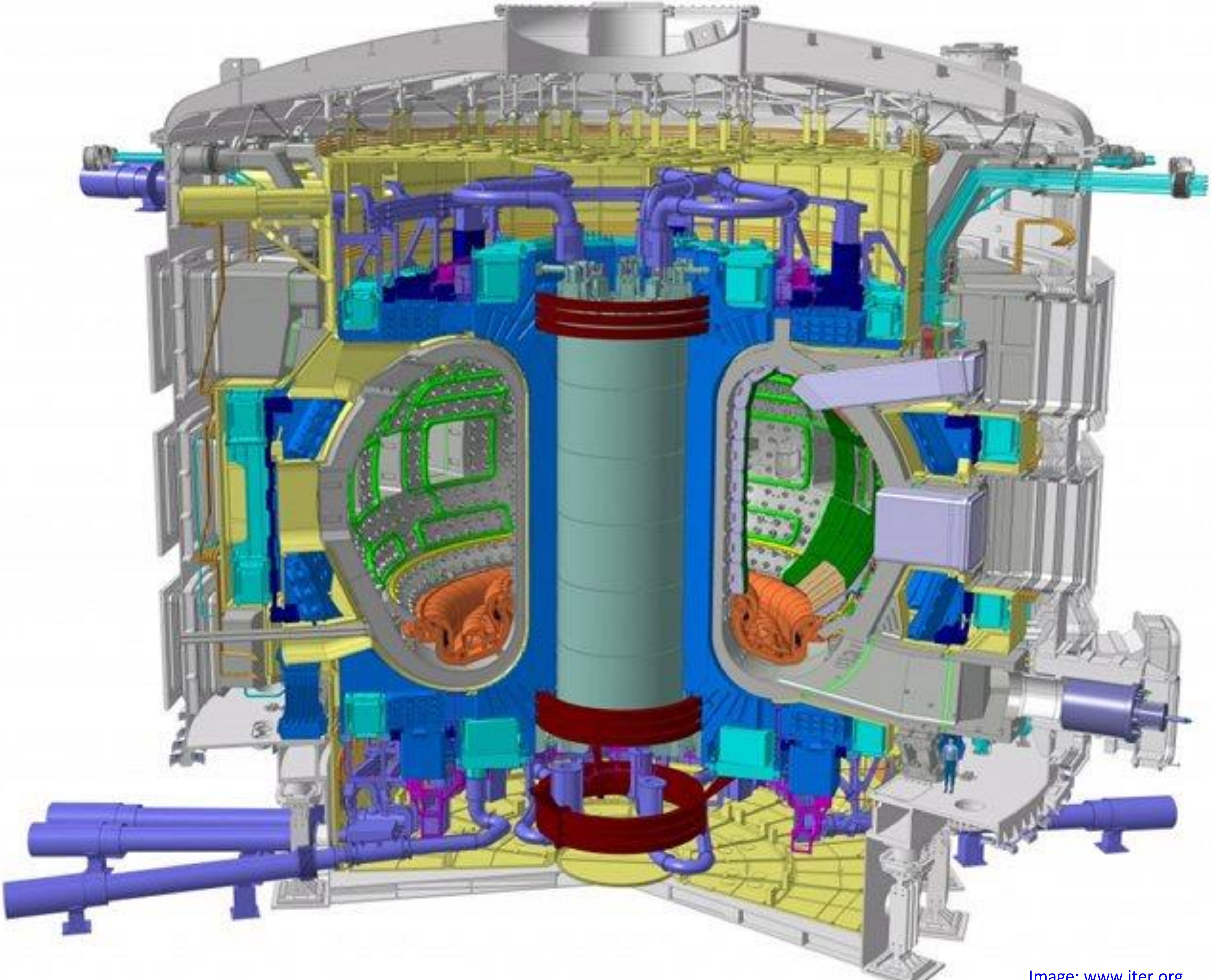

Image: www.iter.org

Cadarache FR, September 2014

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Overview

- **Plasma magnetic control** cascade scheme:
  Inner loop : Vertical Stabilisation (VS)
  Outer loop: plasma Current and Shape Control

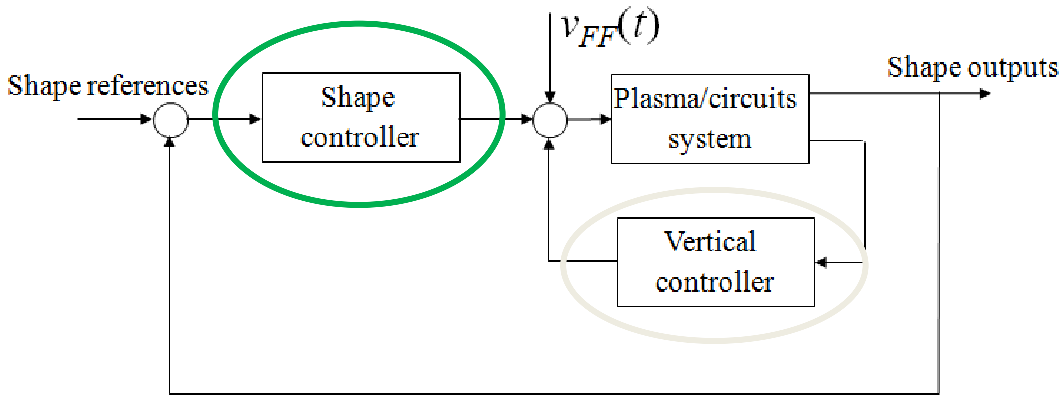
- ITER: A combination of ohmic in-vessel and superconducting poloidal actuators for VS

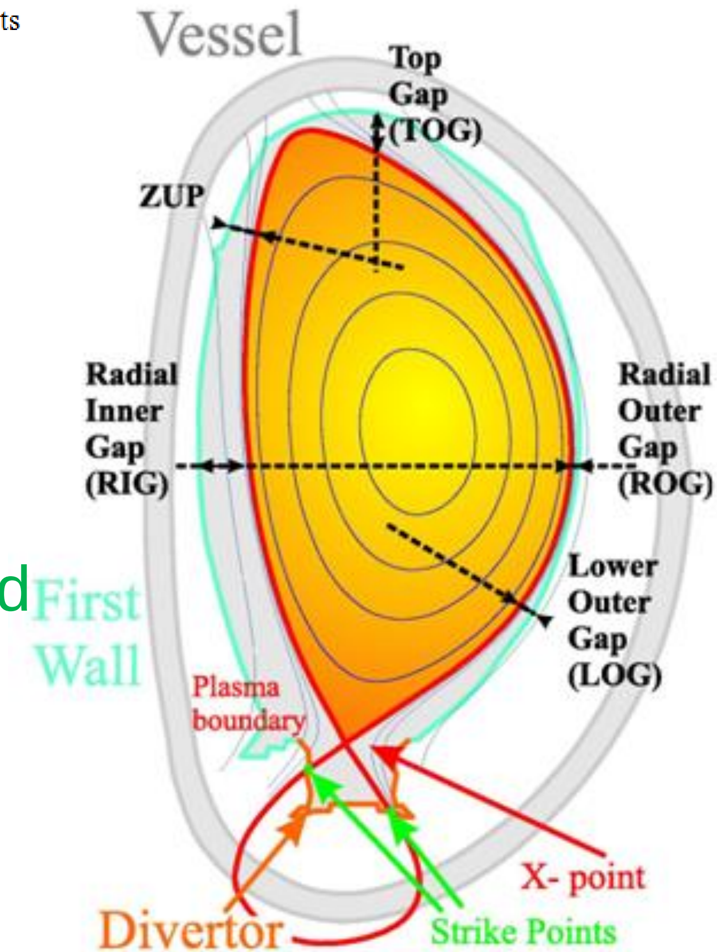
- VS: **ctLQGz** (additional control of plasma vertical position z_p with intermediate dynamics)

- CSC: **Model Predictive Control (MPC)**

- Simulation performance assessment and a feasibility study for implementation

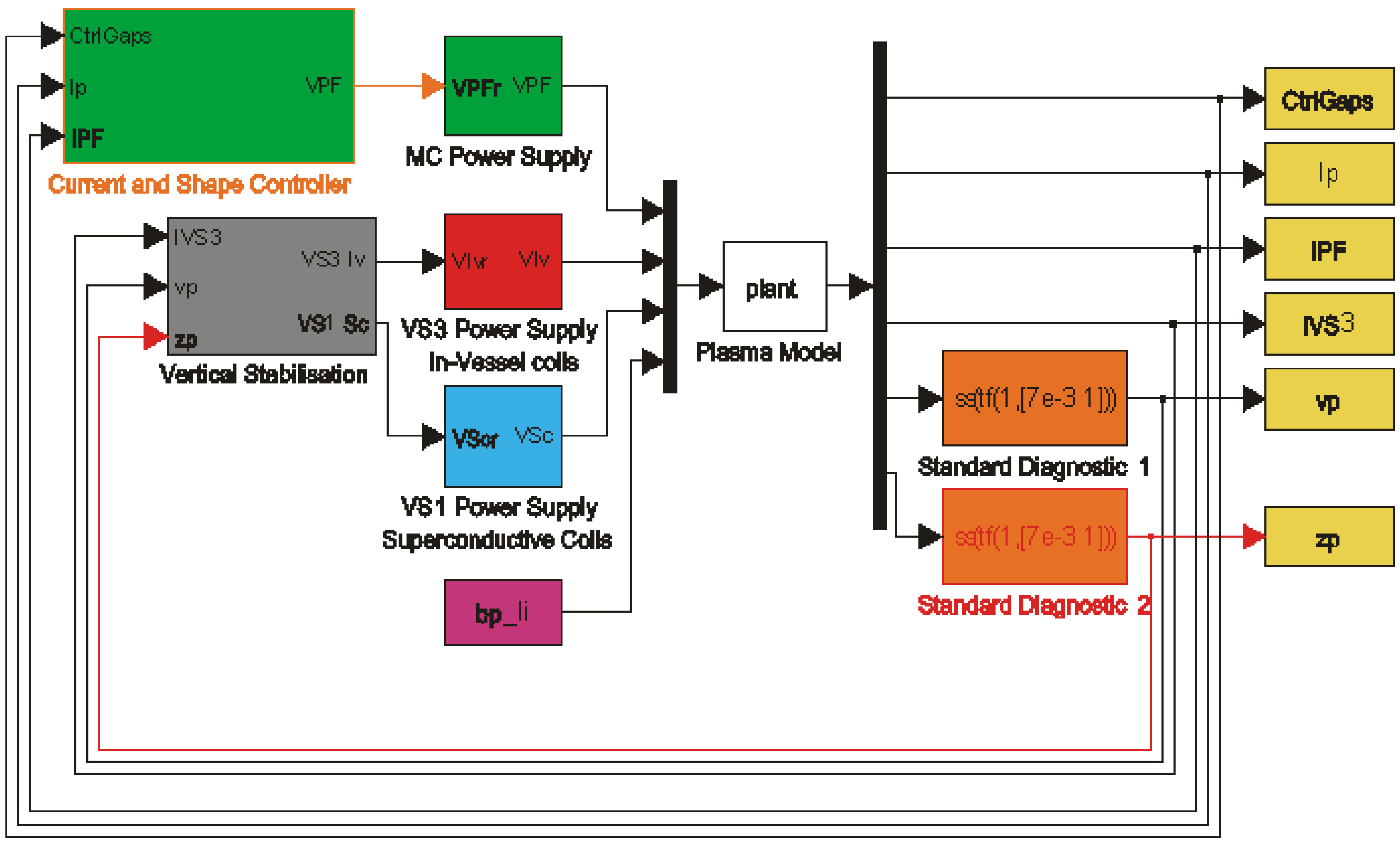
Plasma magnetic control cascade





- Inner loop VS: fast stabilization of vertical position
- Outer loop CSC: plasma current and shape control
- Specific disturbances:
  Vertical Displacement Events
  H-L transitions
  Edge Localised Modes...

Plasma magnetic control scheme with CSC and VS



Plasma simulation models (CREATE-L/-NL)

High-ordel local linear models from first principles

5 models in different equilibrium points of ITER scenarios, defined by the nominal I_{p}, poloidal beta β_p and internal inductance l_i

Simulation of disturbances:

- Vertical displacement event (VDE): via the initial state of the plasma model
- H-L transition: by profiles of β_p and l_i

Model code	I_{p} (MA)	β_p	l_i	Number of states
LMNE	15.0	0.10	1.21	120
LM52	15.0	0.10	0.80	123
LM53	15.0	0.10	1.00	123
LM59	15.0	0.60	0.60	123
LM60	15.0	0.60	0.80	123

Inner loop: VS Vertical Stabilisation

Actuators:

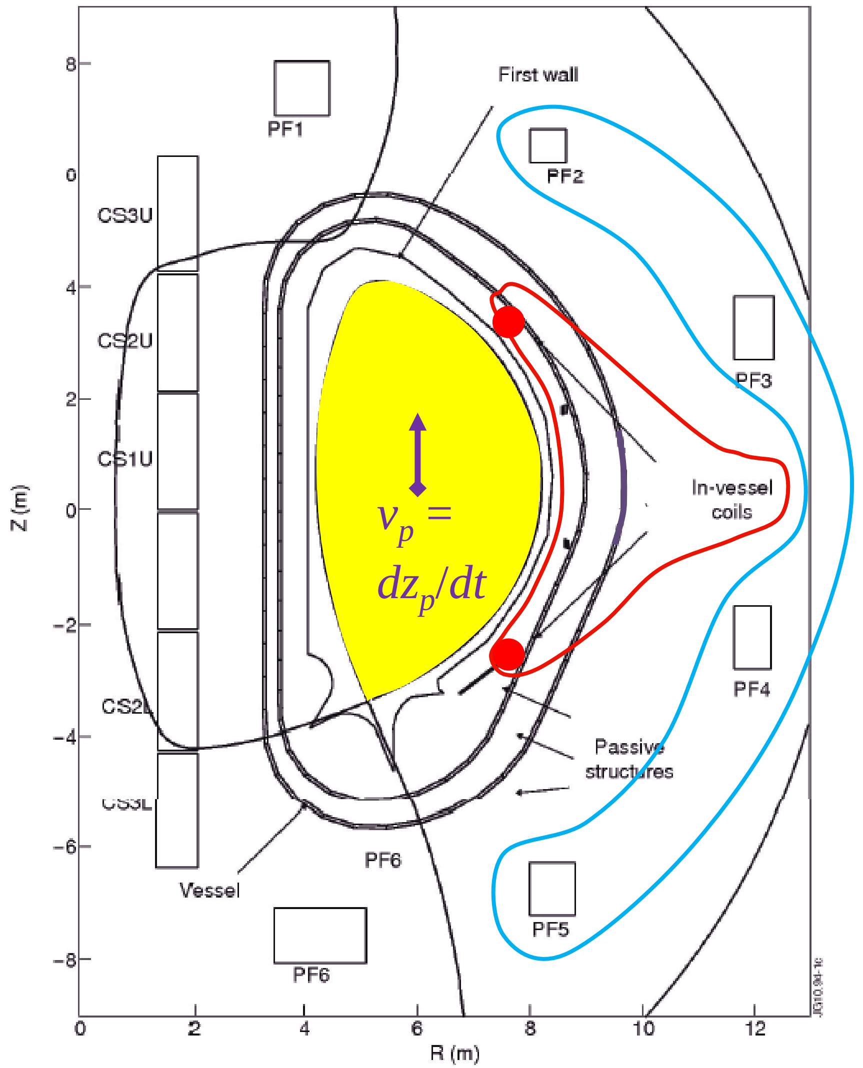
- In-vessel coils (Ic) VS3
  $u_1 = u_{ic}$
- Superconductive (Sc) circuit VS1 (PF2-5) $u_2 = u_{VS1}$

Controlled outputs:

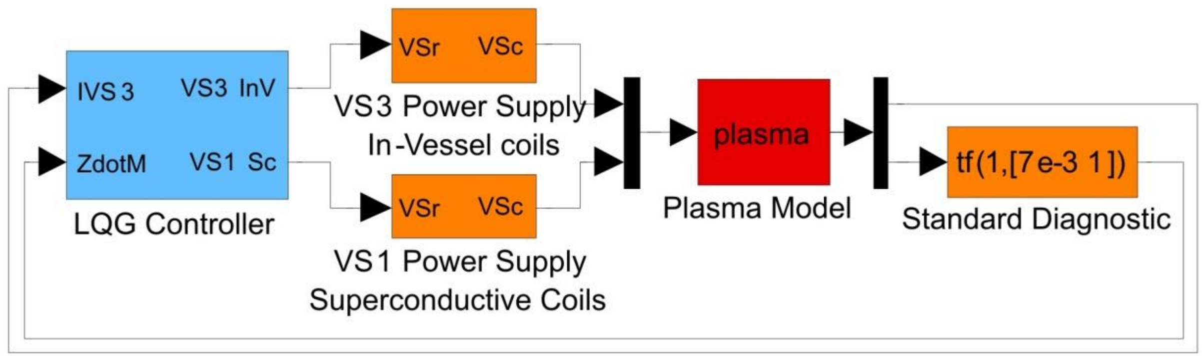
- Plasma vertical velocity
  $y_2 = v_p$
- Ic coils current $y_1 = x_{ic}$
  thermal constraint

Additional ctrl. outputs:

- Plasma vertical position z_p
- Sc circuit current i_{VS1}

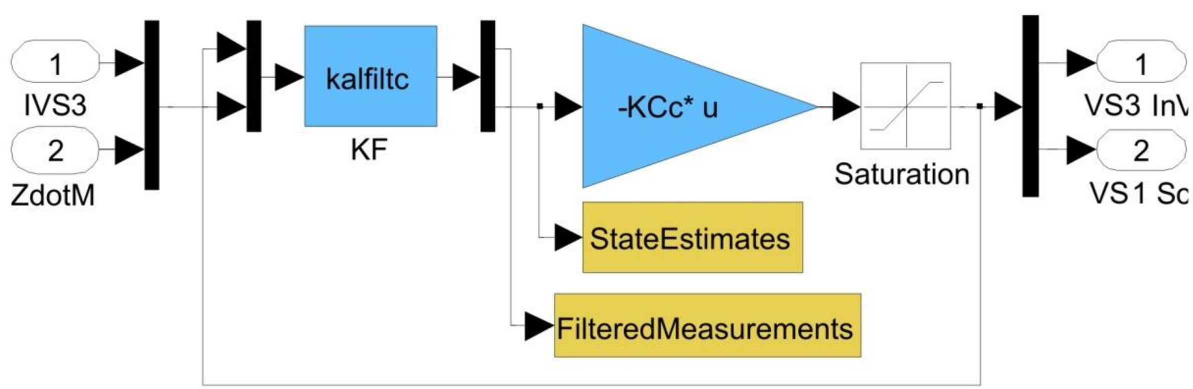


VS: Cont.-time LQG controller (ctLQG)

- Linear-Quadratic optimal controller with Kalman filter (KF)
- Reduced-order model to avoid "over-fitting" to particular local dynamics: Schur balanced truncation (schurmr)
- State **x** not measured; estimated using the KF
- 

LQG block expanded

Saturation: protection against wind-up



VS: ctLQGz = ctLQG + loop from z_p

ctLQG only stops z_p from running away after VDE, relies on CSC to bring it back to the origin

ctLQGz brings z_p back to the origin faster than the CSC would

(SPD+SOF formally relies on freq. separation)

Additonal gain from z_p to VS1 implemented by augmenting the nominal model with an integrator

$$\mathbf{A}_a = \begin{bmatrix} \mathbf{A}_r & \mathbf{0}_{3\times 1} \\ \mathbf{C}_{r,2} & 0 \end{bmatrix}, \quad \mathbf{B}_a = \begin{bmatrix} \mathbf{B}_a \\ \mathbf{0}_{2\times 1} \end{bmatrix}, \quad \mathbf{C}_a = \begin{bmatrix} \mathbf{C}_r & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 3} & 1 \end{bmatrix}$$

Additional tuning parameters

$$\mathbf{Q}_{\mathbf{y}a} = \begin{bmatrix} \mathbf{Q}_{\mathbf{y}a} & \mathbf{0} \\ \mathbf{0} & 2\cdot 10^2 \end{bmatrix}, \quad \mathbf{Q}_{KF,\mathbf{y}a} = \begin{bmatrix} \mathbf{B}_r \mathbf{B}_r^T & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}, \quad \mathbf{R}_{KF,\mathbf{u}a} = \begin{bmatrix} \mathbf{R}_{KF,\mathbf{u}} & \mathbf{0} \\ \mathbf{0} & 10^{-15} \end{bmatrix}$$

Outer loop: CSC Plasma Current and Shape Control

Actuators:

- 11 main power supply voltages $\mathbf{V}_{\mathrm{PF}}$

Controlled outputs:

- Plasma current I_{p}
- 6 controlled gaps $\mathbf{g}$ (2 strike points and 4 gaps)

Additional measured outputs:

- 11 superconductive coil currents $\mathbf{I}_{\mathrm{PF}}$

Singular Perturbation Decomposition (SPD)

A multivariable PI control law from $\mathbf{g}$ and I_{p}, with an additional P contribution from $\mathbf{I}_{\mathrm{PF}}$.

M. Ariola and A. Pironti, An Application of the Singular Perturbation Decomposition to Plasma Position and Shape Control, Eur. J. Control **9** (2003) 410–420

CSC: Model Predictive Control

- Nominal model LM52 preprocessing:
  Append simplified power-supply and sensor dynamics
  VS prestabilisation
  Extract subsystem $\mathbf{u}_{\mathrm{CSC}} = \mathbf{V}_{\mathrm{PF}}$ to $\mathbf{y}_{\mathrm{CSC}} = [\mathbf{I}_{\mathrm{PF}}\ I_{\mathrm{VS3}}\ z_{\mathrm{p}}\ I_{\mathrm{p}}\ \mathbf{g}]^{\mathrm{T}}$
  Model reduction (199 to 44 states)
  Conversion to discrete-time (T_s = 0.1 s, ZOH)
  ...Base model $\{\mathbf{A}_{\mathrm{CSC}}, \mathbf{B}_{\mathrm{CSC}}, \mathbf{C}_{\mathrm{CSC}}, 0\}$
- Control of $\mathbf{g}$ and I_{p} to 0 with integral action,
  (currently without set-point tracking)
- Integral action:
  disturbance-augmentation, 7 integrators at outputs $\mathbf{g}$, I_{p}
- Velocity-form-augmentation to prevent offset when the control signal is non-zero at the steady state:
  $\Delta\mathbf{u}$ becomes the input of the augmented system

Model(-based) Predictive Control

- A control methodology in which future control actions are determined by optimisation of a performance criterion defined over a future horizon in which control signals are predicted using a dynamic process model

- Related to Linear Quadratic optimal control (LQG), they blend in Constrained LQ optimal control

- may handle constraints on process signals, over a finite horizon

- System $\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k), \;\; \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$

- Cost function $J = \sum_{j=0}^{N-1} (\mathbf{x}_{k+j|k}^T \mathbf{Q}_x \mathbf{x}_{k+j|k} + \mathbf{u}_{k+j|k}^T \mathbf{R}_u \mathbf{u}_{k+j|k}) + \mathbf{x}_{k+N|k}^T \mathbf{Q}_{xN} \mathbf{x}_{k+N|k}$

- subject to constraints $\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}, \quad \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{x}_{\max}$

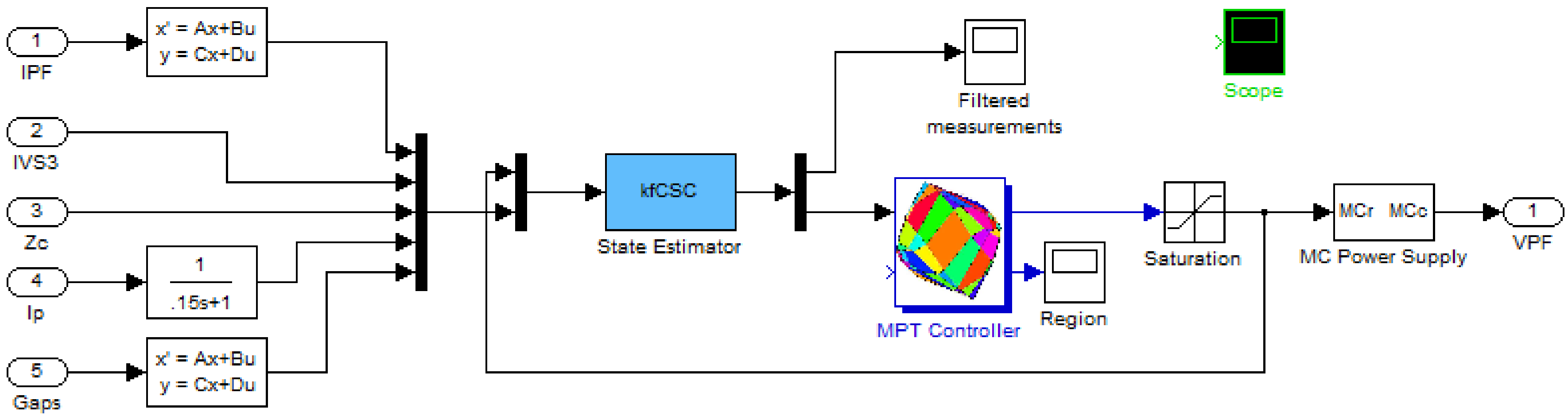
- Receding-horizon implementation

MPC Implementation

- Solved using a Quadratic Programming solver in each step
  QP: min $0.5\, z'Hz + h'z$ subject to $Gz \leq g$, $Fz = f$
- Write down the sequence of predictions over the horizon, form the cost, build the QP matrices
- May be done "manually"
- Matlab MPC Toolbox: configure via menus
  simple and flexible, if everything you need is supported
- Equation parser to build the QP from a problem description
  YALMIP + modified Multi-Parametric Toolbox (or CVX...)

CSC: Model Predictive Control

- MPC is used in an LQG-like scheme where a Kalman filter estimates the states of the disturbance-augmented model.



Simulation comparison

Comparing closed-loop performance of the system using either MPC or SPD as the CSC, and the same VS (ctLQGz)

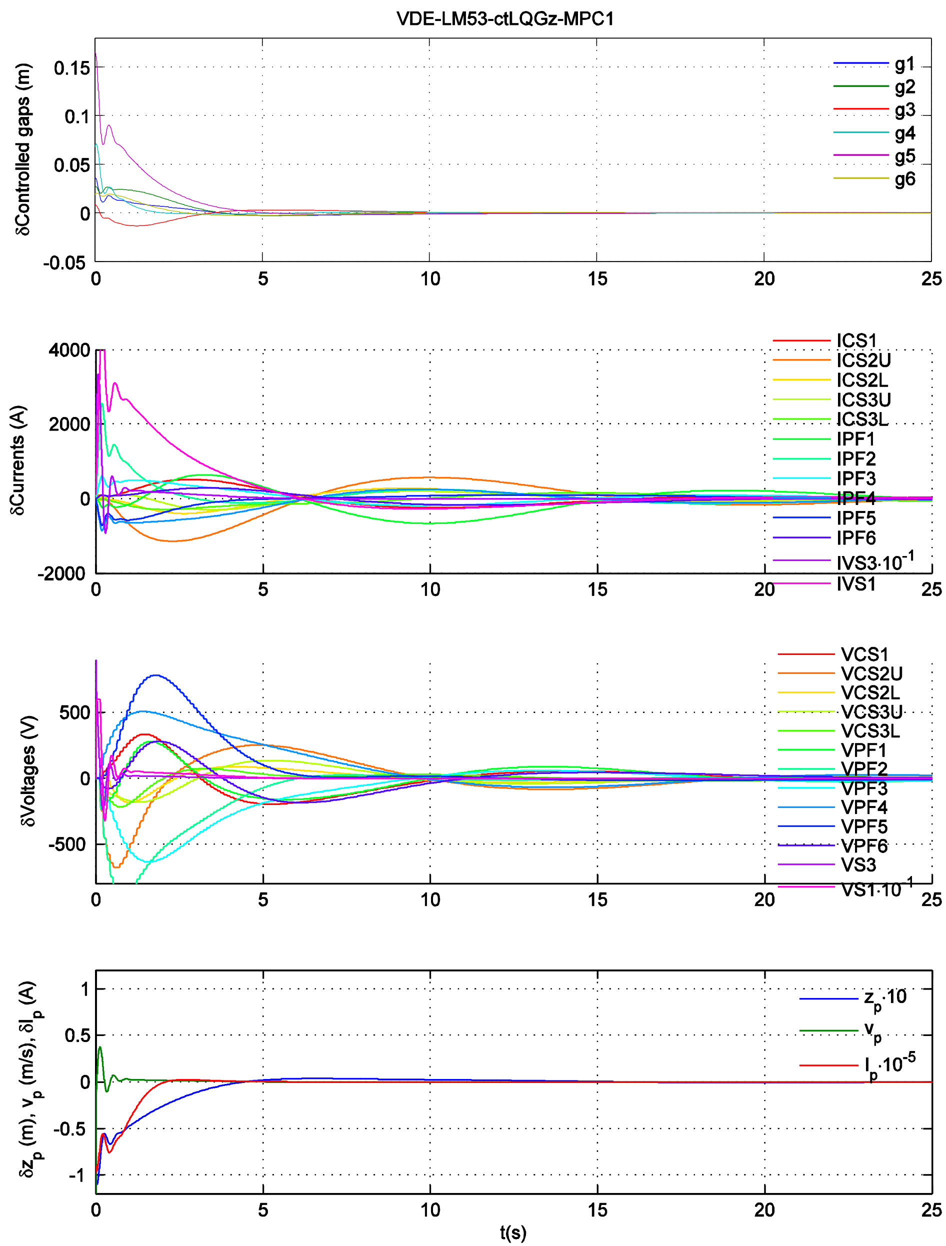
- VDE disturbance, initial amplitude -10 cm
- H-L transition: recorded β_p and l_i profiles ("BPLI") (persistent disturbance)

Tuning parameters chosen so that reasonable responses are obtained with different local models:
LMNE, LM52, LM53, LM59, LM60

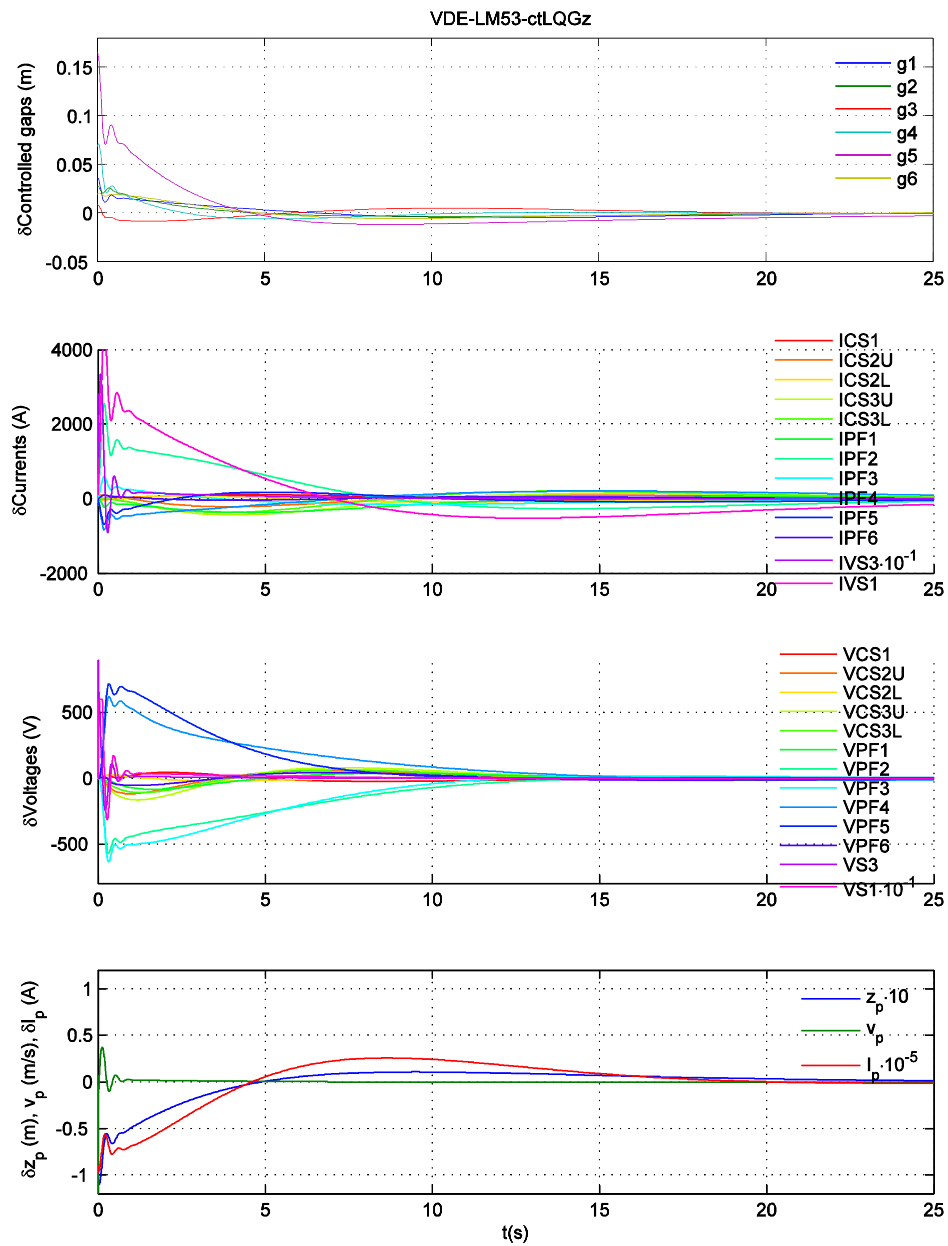
Comparing Root-Integral-Square-Error values (from the equilibria), and graphs of signals visually

Simulation: model LM53, VDE

MPC

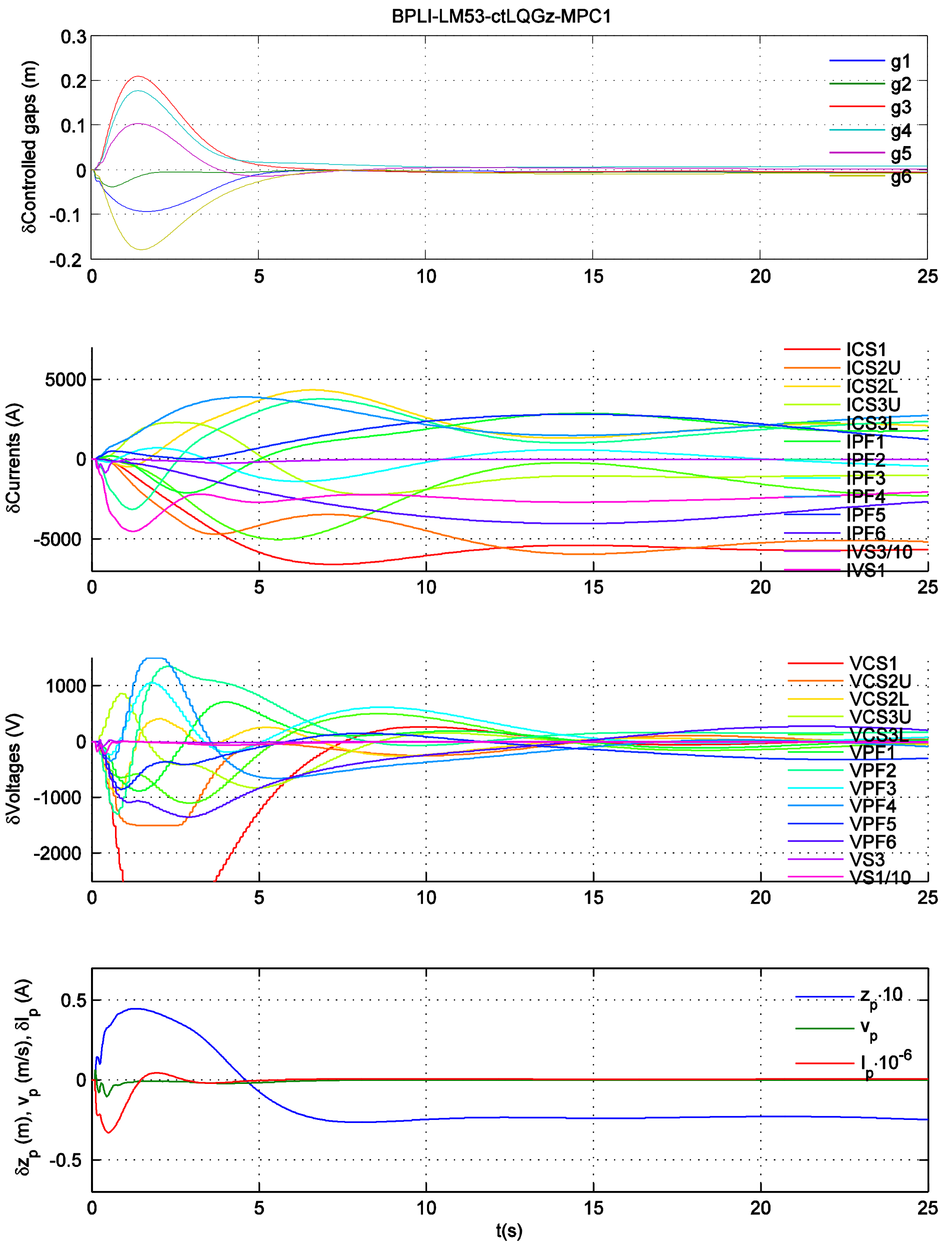


SPD

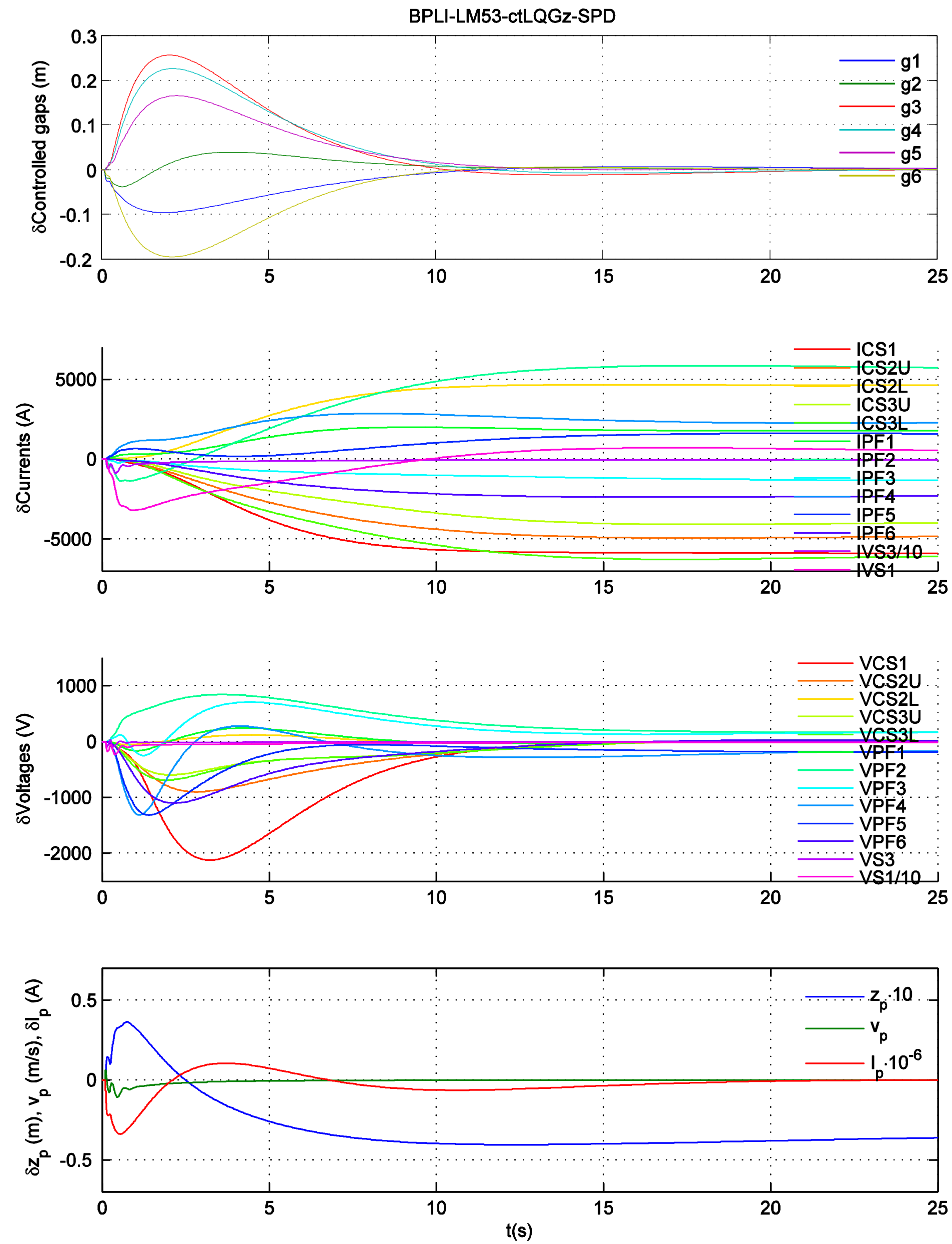


Simulation: model LM53, BPLI

MPC



SPD



MPC performance with constraints

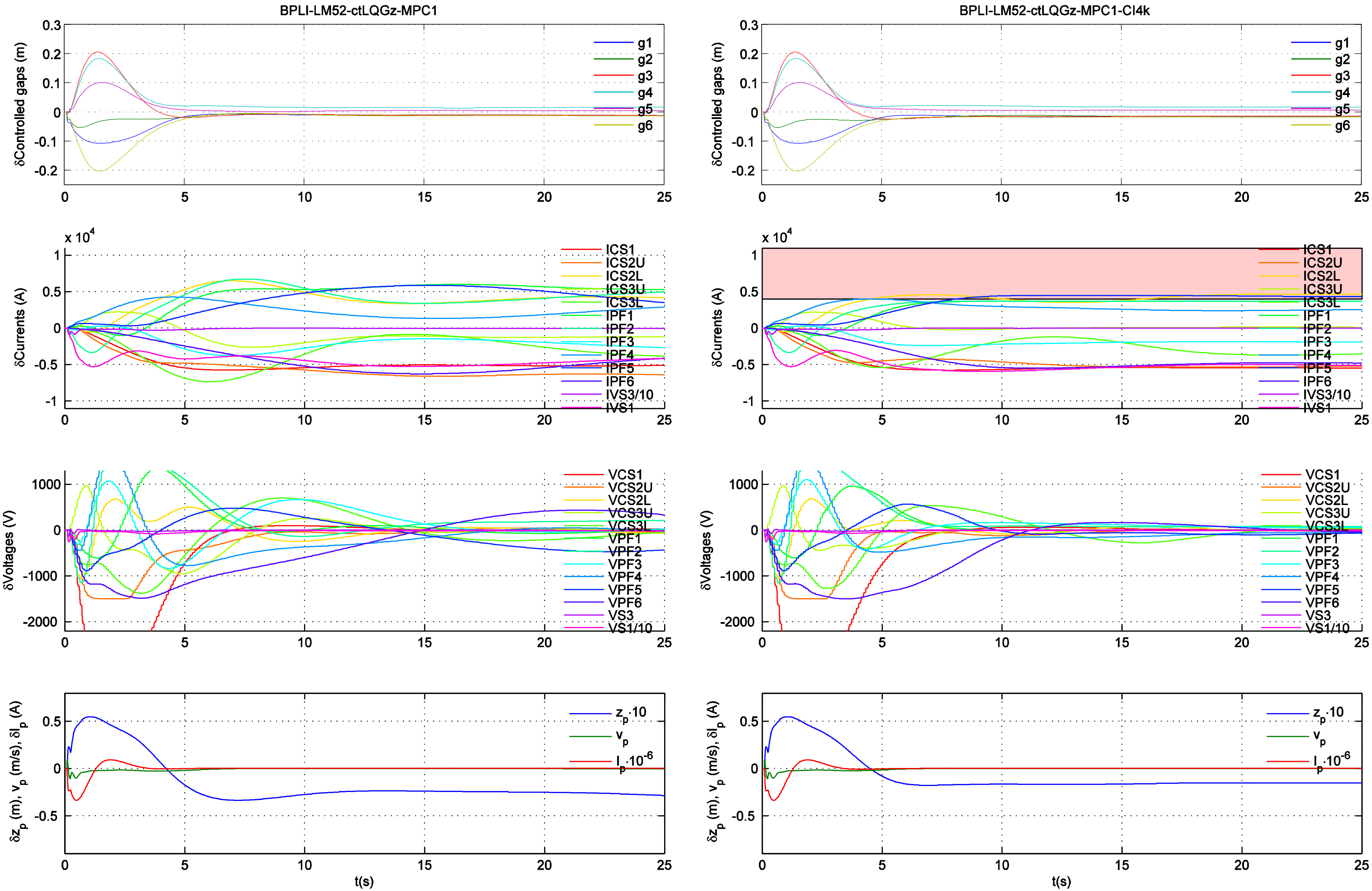
- MPC can consider constraints on control signals (u and y amplitude, u rate...)
- In the example:
  $\mathbf{V}_{PF,min} \leq \mathbf{V}_{PF} \leq \mathbf{V}_{PF,max}$, hard constraints
  $\mathbf{I}_{PF} \leq \mathbf{I}_{PF,\ max}$, soft constraints
  $\mathbf{g} \leq \mathbf{g}_{max}$, soft constraints
- Soft constraints are used at the outputs to avoid infeasibility: Slack variables δ_j are introduced (added to the state)
- Cost: $J = \sum_{j=0}^{N-1} (\mathbf{x}_j^T \mathbf{Q}_x \mathbf{x}_j + \mathbf{u}_j^T \mathbf{R}_u \mathbf{u}_{j|}) + \mathbf{x}_N^T \mathbf{Q}_{xN} \mathbf{x}_N + \sum_{j=0}^{N-1} (\sigma_1 \mathbf{1}^T \delta_j + \sigma_2 \delta_j^T \delta_j)$
- Constraint: $y \leq y_{max} + \delta$
- High σ penalties: optimisation keeps δ at/near zero
  Problem infeasible with a hard constraint...
  soft constraint: QP solution exists but the cstr. not enforced!

MPC performance with constraints

- The peak $\mathbf{I}_{PF}$ currents are **reduced successfully**
  Small violations remain because the constraints are soft
  and because of the offset in the I_{PF} estimate.
- The gap peak is **not reduced**,
  because this controller is tuned tightly
  has no suitable degree of freedom to adjust action.

Simulation: model LM52, BPLI, MPC

No output constraints $I_{PF} < 4$ kA (soft)



MPC Computation

PSC system dimensions: 44 states, 11 inputs, 20 outputs

MPC: horizon 30;

sparse constraints each 5th sample (6 "coincidence points")

input blocking [2 2 26] ... 3*11 = 33 free moves

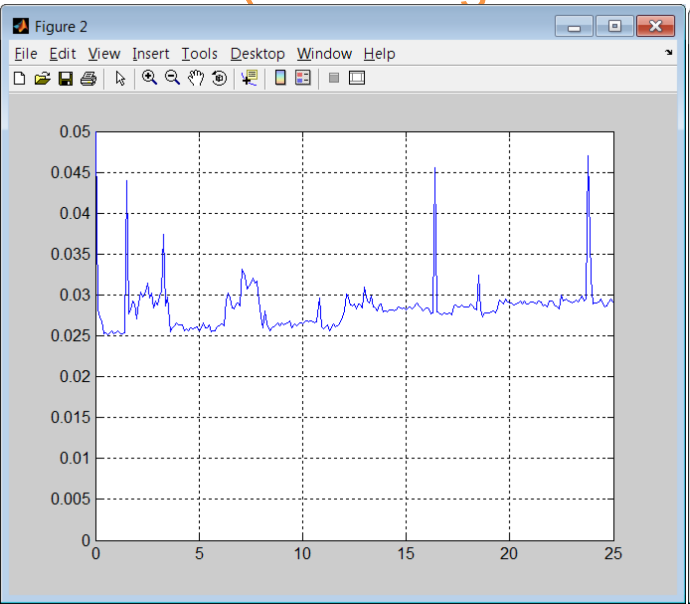
QP generated using MPT/YALMIP ("online controller"):

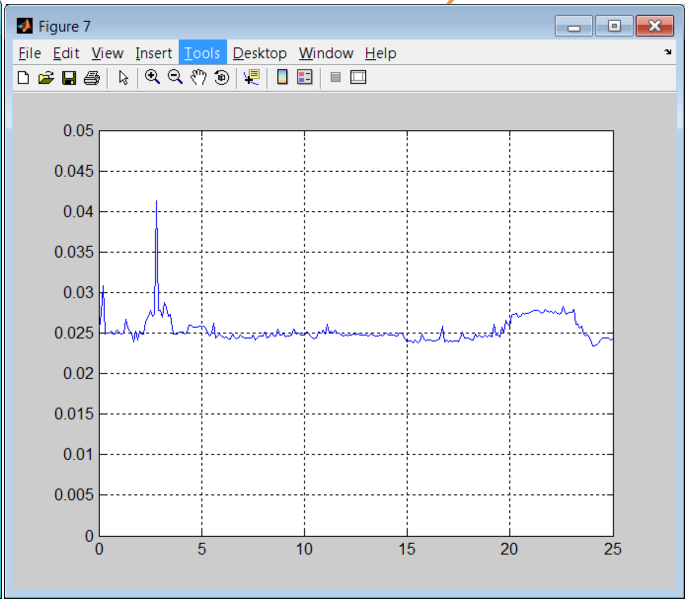
H size 250x250 (sparse, 1306 nonzero entries from 62500)

1968 inequality constraints

CPLEX 11.2 dual-simplex, 1e-9: avg 27 ms, max 54 ms

(including MPT-Simulink overhead)





Conclusions

The feasibility study has shown that efficient simulation performance is achievable using MPC as CSC.

Managing coil-current constraints was demonstrated successfully, without using an intermediate coil-current controller.

This form of MPC is not practically applicable for RT control.

0.1 s sampling appears achievable

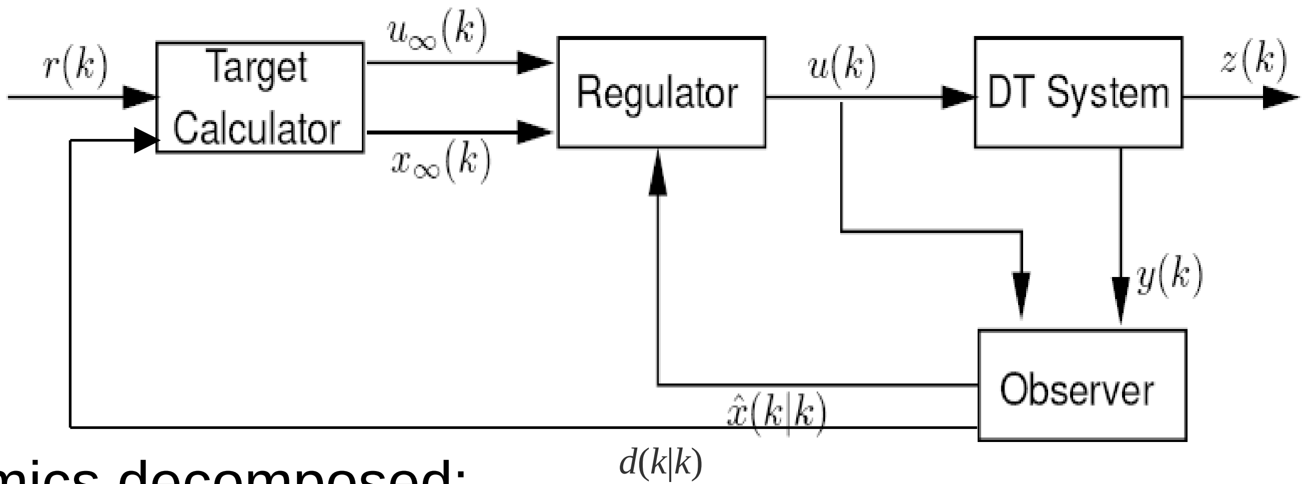
(FMPCFMPC project!)

Upcoming tasks regarding MPC

Target Calculator scheme

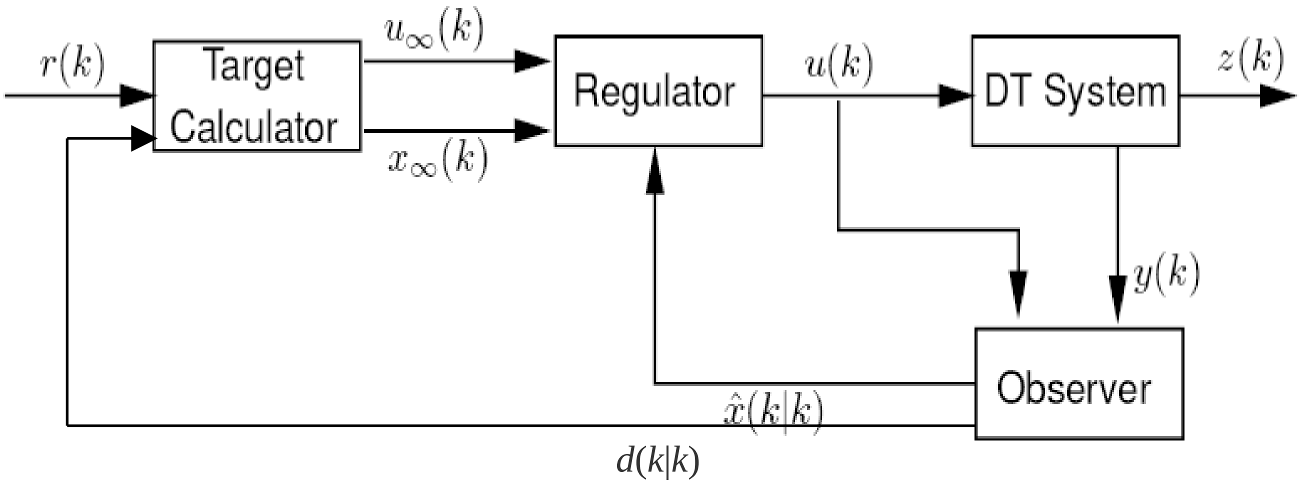
Fast MPC implementation

Target Calculator scheme



- Process dynamics decomposed:
  steady-state (Target Calculator), transient (Dynamic Controller)
- The concept related to the "Current Limit Avoidance" scheme
- Origin in practice, infeasibilities: finding a feasible steady state is the most important, transient violations matter less
- Useful suboptimal practices:
  TC optimisation problem is reduced;
  DC: an unconstrained LS or a LQ solution may be useful (but does not actively reduce transient constraints violations like MPC)
- The Estimator is made for the whole system
- The TC+KF is not entirely a steady-state affair, provides I control

Target Calculator scheme



- The DC controls the state and input to the origin,
  so infinite-horizon MPC may be used
  (with more CVs than actuators, SVD needed)
  (not fair to ignore the TC in the system though)
- An optimal solution should compute both the steady state and dynamic control at once
- Stability theory available recently (but not with a QP solver)

Zeilinger Morari Jones: 'Soft constrained model predictive control with robust stability guarantees',
IEEE Transactions on Automatic Control, 59(5) (2014) 1190-1202
TC application with fast MPC: Hartley Jerez Suardi Maciejowski Kerrigan Constantinides 2014
Predictive Control Using an FPGA With Application to Aircraft Control, IEEETCST 22(3) 1006

Fast online MPC

MPC typically translates to Quadratic Programming:
quadratic cost function with linear inequality constraints

Boyd Vandenberghe 2004 **Convex Optimisation**

Book: http://stanford.edu/~boyd/cvxbook/bv_cvxbook.pdf
Slides: http://stanford.edu/~boyd/cvxbook/bv_cvxslides.pdf
Software – CVX, CVXGEN: http://stanford.edu/~boyd/software.html

QP: min 0.5 z'Qz + q'z (MPC cost) subject to

inequality constraints $Gz \leq g$ (actuator & state constraints)

equality constraints $Fz = f$ (process dynamics)

(eq.c. sometimes eliminated... structured vs condensed QP)

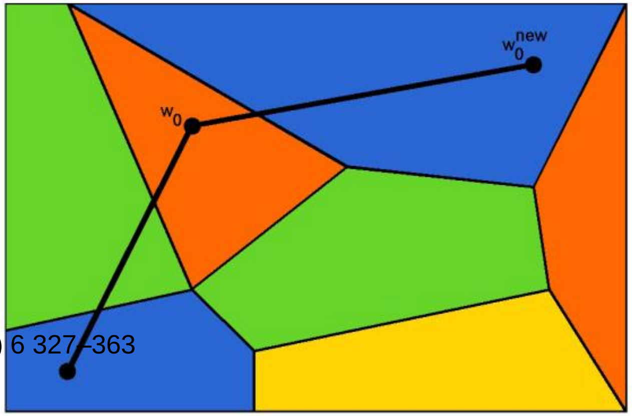
On-line QP solvers

- Active set methods
- Interior point methods
- First order methods: don't require solving a system of eqs each iter.!
  slower convergence, but faster at the required precision
  (while some use IP or AS methods with a MINRES solver)

Fast online MPC

Active set QP methods

- Find active set of inequality constraints at solution through iterations
- Related to explicit MPC: each working set – one polyhedral region
- No of combinations typically prohibitive for a brute-force approach
- Each iteration removes/adds one constraint (entering/leaving), requires solving a system of equations
- Typically very fast convergence, but longer near constraints
- Upper bound of iterations said to exist but not useful practically
- Matlab Opt. Tbx: not reliable numerically; CPLEX, NAG etc
- Real-Time version qpOASES:
  parametric active-set QP
  open-source, not most popular lately



https://projects.coin-or.org/qpOASES
Ferreau Bock Diehl 2008
An online active set strategy to overcome the limitations of explicit MPC, IJRNC 18(8)
Ferreau Kirches Potschka Bock Diehl 2014
qpOASES, a parametric active-set algorithm for quadratic programming, Math. Prog. Comp. (2014) 6 327–363

Fast online MPC

Interior-point QP methods

- Approach the solution by traversing the interior of the feasible region usually employing a barrier function
  (interior penalty, smooth and strongly convex)
  ... replace a hard constraint with a smooth logarithmic barrier
- Then, basically any method for smooth convex unconstrained minimization can be used, e.g., the Newton method
- solving a system of KKT equations in each iteration

Fast online MPC

Interior-point QP methods

- **Wang Boyd 2010 Fast MPC Using Online Optimization**, IEEE TCST 18(2)

Primal barrier IP method:

a barrier term (log function of ineq constraints) is added to the QP:

min z'Hz + g'z + $\kappa\phi(z)$ subject to Cz = B

Infeasible start: z_0 satisfies only ineq constraints; eq.c. converge

6-oscillating-masses benchmark, 12 states, 3 controls, hard cstr;

Speed-up vs regular (SDPT3) from 3.4 s to 26 ms (at horizon 30) using a CPU, mainly by structure sparsity exploitation

With longer horizons, exploiting special MPC structure works better than a condensed formulation

Early termination (5 iterations)... warm starting helps considerably

MPC Performance degradation "barely noticeable"

Precursor to CVXGEN

Fast online MPC

Interior-point QP methods

- **Mattingley Boyd 2012**
  CVXGEN a code generator for embedded convex optimization, Optim Eng (2012) 13 1–27
  Mattingley Wang Boyd 2011
  Receding Horizon Control, Automatic Generation of High-Speed Solvers, IEEE CSM 31

  CVXGEN Translates a "disciplined" description of a convex optimisation problem to optimised library-free C code suitable for embedded applications

  Fast and robust solver (should not fail with imperfect data)

  Limited accuracy

  Each instance optimised for a problem family and HW platform

  Primal-Dual Interior Point QP method

  Solving the KKT system of equations:

  LDL factorisation,regularisation,

  iterative refinement, dynamic regularisation

  MPC example against CVX/SeDuMi

Fast online MPC

Interior-point QP methods

- **Huang Ling See 2011 Solving QP problems on GPU**, ASEAN Engineering Journal Vol.1 No.2

  Relatively early, "odd-ball", not best paper;

  Discusses GPU (CUDA);

  Speed-up by splitting a matrix-vector multiplication among cores

  Solving the system of eqs done sequentially on CPU (long division)!

Fast online MPC

Interior-point QP methods

- **Domahidi Zgraggen Zeilinger Morari Jones 2012**
  Efficient Interior Point Methods for Multistage Problems Arising in RHC, CDC12

  Primal-Dual Interior Point QP solver FORCES

  Solves the KKT system of eqs using Cholesky factorisation

  Able to handle larger problems than CVXGEN

  Also supports quadratic constraints (QCQP) and second-order cone programs (SOCP), required by some MPC methods

  2-5 times faster than CVXGEN, 10-100 times faster than CPLEX

  Benchmark BP1: M-oscillating-masses, hard output constraints
  the largest: 60 states, 29 inputs, horizon 30:
  PC Core i7: CVXGEN 130 ms, CPLEX 180 ms
  Atom Z530: 1s; ARM Cortex V8: 40 s

  BP2: + quadratic terminal cost, quadratic terminal constraint, constraint ensuring stability with early termination (QCQP)
  PC: 220 ms

  Used to be a free code-generation web service, forces.ethz.ch/

Fast online MPC

Interior-point QP methods

- **Frison Kufoalor Imsland Jorgensen 2014, IEEE CCA 2014 Antibes FR**
  Efficient Implementation of Solvers for Linear MPC on Embedded Devices

Embedded platforms: Intel Atom, ARM Cortex A9, PowerPC 603e
  (single-core only)
Interior-Point QP solver HPMPC
  bottleneck: search direction computation – Riccati iterations
Optimizing linear algebra operations (matrix-matrix multiplication)
  to reduce memory movements (memop takes more than flop)
SIMD: Atom and A9 both 4 floats wide (SSE / NEON),
  highly processor-specific!
Benchmark: M-oscillating-masses, hard output constraints
the largest: 60 states, 29 inputs, horizon 30
  HPMPC 0.3 s vs FORCES 1.5 s

Fast online MPC

Proximal Newton methods

Related to the recently popular group of first-order methods

- **Patrinos Bemporad 2013**
  Proximal Newton Methods for Convex Composite Optimization, CDC2013
- **Guiggiani Patrinos Bemporad 2014**
  Fixed-Point Implementation of a Proximal Newton Method for Embedded MPC, IFAC WC 2014
- **Patrinos Stella Bemporad s.2014 Forward-backward truncated Newton methods for convex composite optimization**

Fast online MPC

First order (gradient, ascent) QP methods

- Approach the solution of KKT optimality conditions by succesive gradient descent steps, don't need to solve a system of eqs.
- Origin: Nesterov 1983, then not much attention for a long while: Slow convergence... lots of iterations needed for high precision
- Interesting for control, because they're "lightweight" and therefore efficient in achieving low precision when sufficient
  May be customised for MPC problems
  Very short computation times on single core CPU already
  Suited to FPGA etc due to relative simplicity (resource bounded!)
  (CPU: matrix-vector multiplication... bounded by memory access)
- The algorithm involves a projection, generally as hard as a QP itself
  Simple with simple bounds on control inputs
  Not as simple with state constraints
  (computation; convergence speed)

Fast online MPC

First order (gradient, ascent/descent) QP methods

- **Jerez Goulart Richter Constantinides Kerrigan Morari 2014**
  Embedded Online Optimization for MPC at Megahertz Rates, IEEE TAC 59(12)

Fast Gradient Method (input-constrained problems only)

Future states eliminated, expressed as function of the initial state

$\min_z 0.5\, z'Hz + z'\Phi x$, $z = (u_0, \ldots u_{N-1})$ (condensed format)

Iterations involve a matrix-vector multiplication and a projection

State cstr: dual fn not strictly concave, sub-linear convergence

Alternate Direction Method of Multipliers (state-constrained too)

(soft constraints: quadratic and linear cost... exact penalty)

$\min_z 0.5\, z'Hz + z'h$, $z = (u_0, \ldots u_{N-1}, x_0, \ldots x_N)$ (non-condensed)

subject to $Fz = b(x)$ (state update equation)

ADMM partitions the optimisation variables into two groups

to maintain the possibility of decoupled projection... y a copy of z

Slow convergence with soft constraints... rescaling

Fast online MPC

First order (gradient, ascent) QP methods

- **Jerez Goulart Richter Constantinides Kerrigan Morari 2014**
  Embedded Online Optimization for MPC at Megahertz Rates, IEEE TAC 59(12) **(cont.)**
  FPGA implementation in fixed-point arithmetic
- Overflow errors: upper&lower bounds on all variables needed
  ADMM: upper bound on the Lagrange multiplier not available
- Arithmetic round-off errors (multiplication: truncation), quantisation:
  Establish a converging upper bound on the total incurred error
  ...it is possible to determine the required no of bits
  Normalisation of H so that max eigenvalue is less than 1
- Benchmark: 4-oscillating-masses, 4 inputs 8 states
  No disturbance model (?)
  FGM: input bounds only, 15 iterations, Ts $\approx$ 1 μs
  ADMM: also soft state constraints, 40 iterations, Ts $\approx$ 10 μs
  (depends on the degree of parallelisation: bulkier code, less par.)
  FORCES PRO https://www.embotech.com/FORCES-Pro

Fast online MPC

First order (gradient, ascent) QP methods

- **Richter 2012 FiOrdOs, Code Generation for First-Order Methods, ISPM 2012 Berlin**

  MSc thesis project of F. Ullmann; S. Richter ETH Zurich

  Appears to be related to FORCES Pro

  Fast Gradient Method

  In case of equality and/or inequality constraints:

  Lagrange relaxation or Primal-dual approach with preconditioning

  Matlab toolbox for C code generation (incl. MEX and Simulink), open source

Fast online MPC

First order (gradient, ascent) QP methods

- **Peyrl Zanarini Besselmann Liu Boéchat (ABB) 2014**
  Parallel implementations of the fast gradient method for high-speed MPC, CEP 33
  Fast Gradient Method, MPC with input bounds only
  Required precision & overflow bounds analysis
  Benchmark: 2 to 16 oscillating masses
  Implementation, in curious details
  FPGA (Cyclone V)
  Multi-core CPU (PowerPC Freescale P4080, no SIMD)
  3 masses, 15 vars, 0.1%: FPGA 0.3 μs, CPU 10 μs

Fast online MPC

First order (gradient, ascent) QP methods

- **Patrinos Bemporad 2014,** IEEE TAC 59 (CDC 2012)
  An Accelerated Dual Gradient-Projection Algorithm for Embedded Linear MPC
  Accelerated Dual Gradient-Projection (GPAD) method
  Fast, simple, small memory footprint, short worst-case time, certifiable
  General polyhedral constraints on inputs and states
  FGM is applied to the dual problem, resulting from relaxing ineq cstrs
  Convergence bounds for dual and primal optimality, primal feasibility
  Pre-specified accuracy... determine worst-case number of iterations
  Ball-and-plate example
  Oscillating Masses example
  Compared to qpOASES and a bunch of AS&IP solvers (not other first-order methods)
  Sensitivity to scaling – preconditioning important
  Fixed-point implementation for FPGA...
  Patrinos Guiggiani Bemporad 2013 Fixed-Point Dual Gradient Projection for Embedded MPC, ECC2013

Fast online MPC

First order (gradient, ascent) QP methods

- **Rubagotti Patrinos Bemporad 2014**
  Stabilizing Linear MPC Under Inexact Numerical Optimization, IEEE TAC 59
  Stability with GPAD in real-time case with
    early termination when the solution is suboptimal,
    inequality constraints are not satisfied exactly
  Primal methods: a suboptimal solution does not violate ineq.c.
  Dual (applicable to more general problems):
    inexact solution to the primal problem
- Take the tolerances in account in the MPC formulation
  Ensure asymptotic stability with bounded performance loss

Fast online MPC

First order (gradient, ascent) QP methods

- **Giselsson 2015 Improving Fast Dual Ascent for MPC - Part II The Embedded Case, arxiv.org abs 1312.3013 v2(Automatica)**

  Genaralizes Fast Dual Gradient Method and Alternating Direction Method of Multipliers to achieve faster convergence

  More general curvature of the quadratic upper bound

  Alg. #1 (gen. FDGM Richter 2013)

  Alg. #2 (gen. FDGM Patrinos&Bemporad 2014,
  ADMM O'Donoghue&al 2013)

  AFTI-16 benchmark, 4 states 2 inputs 2 outputs, precision 0.5%
  soft output constraints (quadratic penalty only)

  Single-core CPU Matlab: max 10 ms, ca 100x faster
  (not clear if due to the particularly difficult problem)

  C-code: max 0.2ms, 3x faster than FORCES (IP), 20x than MOSEK

  Alg #1 does not support soft cstr... a mpQP with 2 regions used

  Alg #2 does; projection also parametrically, 1 max operator only
- Giselsson Boyd 2015 Metric Selection in Douglas Rachford Splitting and ADMM. Submitted.
- Giselsson Boyd 2015 Metric Selection in Fast Dual Forward Backward Splitting. Submitted.

Fast online MPC

Some applications and reviews

- **Hartley Jerez Suardi Maciejowski Kerrigan Constantinides 2014**
  Predictive Control Using an FPGA With Application to Aircraft Control, IEEETCST 22(3)
  Boeing 747-200 benchmark with many manipulated signals
  FPGA Xilinx V6-LX240T 250MHz, FORCES
  Target Calculator scheme:
- MPC Dynamic Controller:
  12 states, 17 inputs, 10 disturbances, Ts 0.2 s
  no output constraints
  Primal-Dual Interior Point QP solver, fixed at 18 iterations
  Parallel MINRES with offline prescaling and online preconditioning
  for solving the system of eqs
  Single-precision floating point
  FORCES: horizon 12: FPGA 12 ms, PC 13 ms (CVXGEN hor. 5 max)
- Target Calculator:
  Fast Gradient Method
  ("dense" QP not MPC structure; constraints are simple bounds)
  fixed-point... to save FPGA resources

Fast online MPC

Some applications and reviews

- **Stathopoulos Szucs Jones 2014 Splitting methods in control, ECC2014**
  Alternating Direction Method of Multipliers (ADMM)
  Alternating Minimization Algorithm (AMA)
  Primal-dual scheme of Chambolle and Pock (CP)
  generalized to Proximal ADMM, Generalized ADMM or whatever
  Boeing 747-200 benchmark 12 states 17 inputs,
  Target Calculator and Dynamic MPC subproblems
  TC: ADMM 0.56 ms, DC: FAMA 43 ms (platform??)

Fast online MPC

Some applications and reviews

- **Kufoalor Richter Imsland Johansen Morari Eikrem 2014, MED 2014 Palermo Embedded MPC on a PLC Using a Primal-Dual First-Order Method for a Subsea Separation Process**

  Subsea separation process (Statoil):
  4 CVs, 3 MVs, 6 move-blocking indices, 2 MDs, 6 slacks, horizon 10
  58 eq cstr, 138 ineq cstr, 82 decision vars
  QP not strictly convex (perturbed H used with some methods)
  MPC: SEPTIC, MIMO FSR model, OSD ("industrial")
  PLC platform: ABB AC500 (library-free C code; single-core CPU)
  Primal-dual first-order method FiOrdOs
  Projection operation on the output constraints is not simple,
  a multi-parametric solution (MPT) yields 30000 regions...
  output inequalities kept as inequalities
  Preconditioned primal-dual first-order method
  (Chambolle Pock, FiOrdOs pre-release)
  Better than 5 recent first-order methods (some using CPLEX proj.)
  and PDIP (CVXGEN)

Fast online MPC

Fast online MPC with soft constraints

- **Kufoalor Richter Imsland Johansen Morari Eikrem 2014, MED 2014 Palermo**
  Embedded MPC on a PLC Using a Primal-Dual First-Order Method for a Subsea Separation Process
- **Jerez Goulart Richter Constantinides Kerrigan Morari 2014, IEEE TAC 59(12)**
  Embedded Online Optimization for Model Predictive Control at Megahertz Rates
- **Zeilinger Morari Jones 2014, IEEETAC 59(5)**
  Soft Constrained Model Predictive Control With Robust Stability Guarantees