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EUROfusion

MPC for Plasma Magnetic Control

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SLOVENIAN RESEARCH AGENCY

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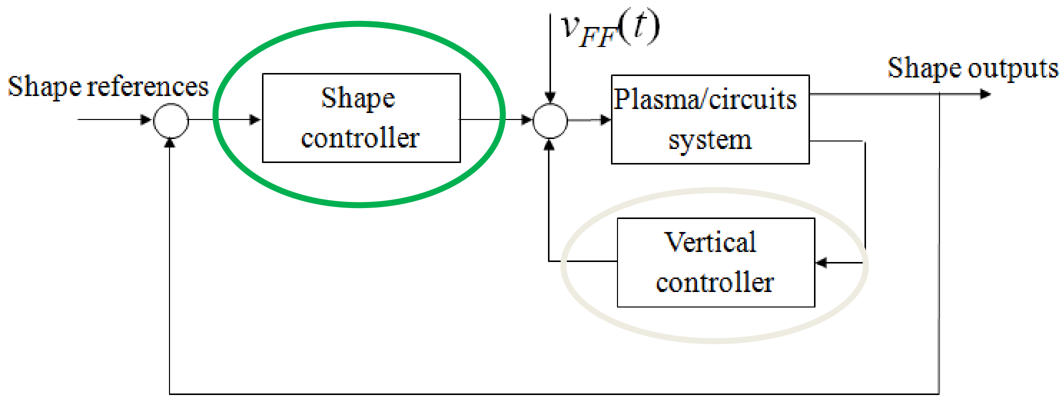
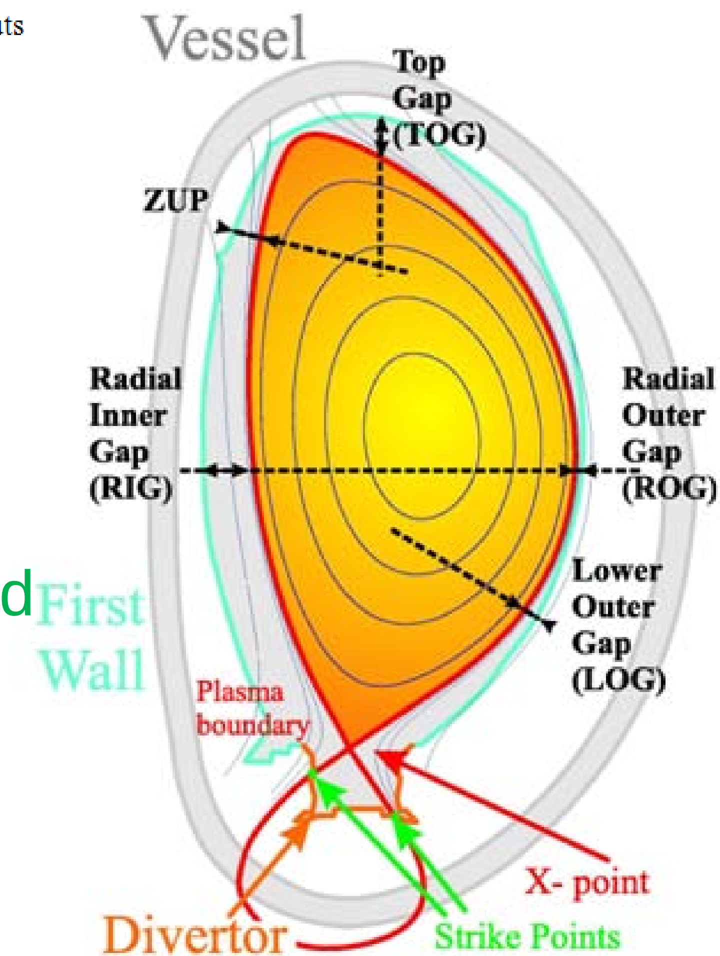
The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Overview

- **Plasma magnetic control** cascade scheme:
  Inner loop : Vertical Stabilisation (VS)
  Outer loop: plasma Current and Shape Control
- ITER: A combination of ohmic in-vessel and superconducting poloidal actuators for VS
- VS: the same as in CREATE v2d0 scheme:
  based on Static Output Feedback (in fact dynamic)
- CSC: **Model Predictive Control (MPC)**

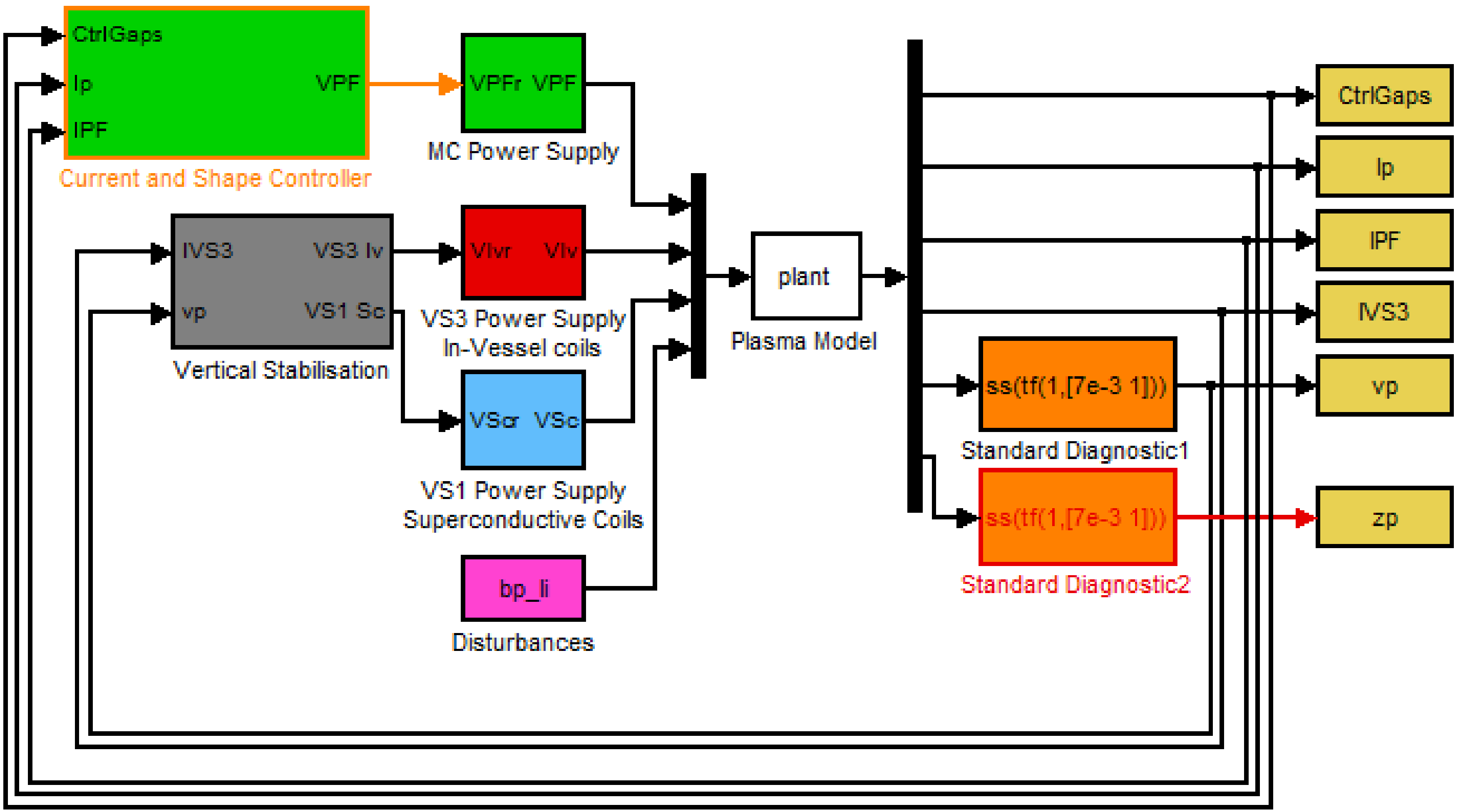
Plasma magnetic control cascade





- Inner loop VS: fast stabilization of vertical position
- Outer loop CSC: plasma current and shape control
- Specific disturbances:
  Vertical Displacement Events
  H-L transitions
  Edge Localised Modes...

Plasma magnetic control scheme with CSC and VS



Plasma simulation models (CREATE-L/-NL)

High-ordel local linear models from first principles
(~120 states)

14 models in different equilibrium points of ITER Scenario 1,
defined by the nominal I_{p}, poloidal beta β_p and internal inductance l_i

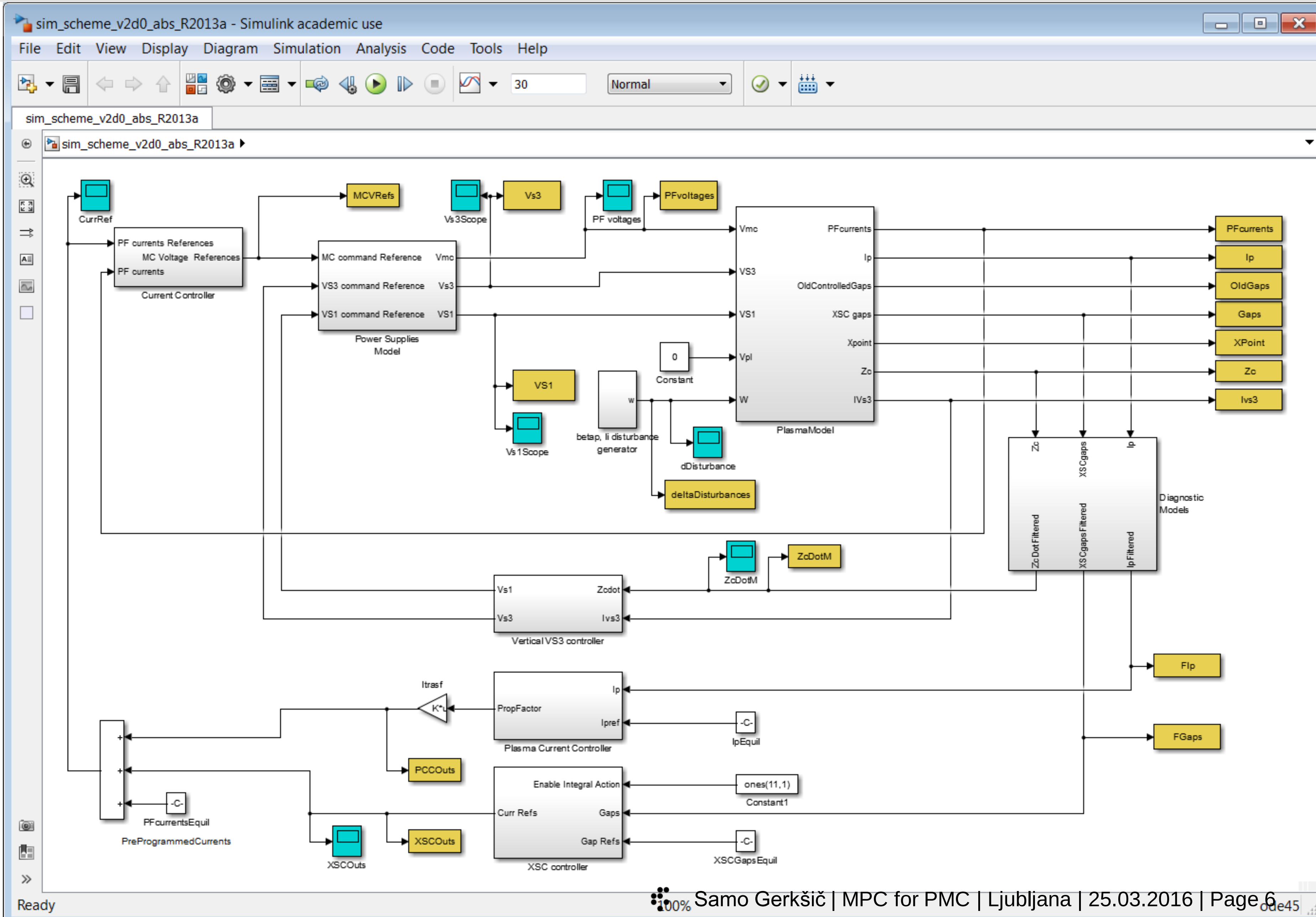
Simulation of disturbances:

- Minor disruption, Uncontrolled ELM, L-H transition, H-L transition: by profiles of β_p and l_i inputs
- Vertical displacement event (VDE): via the initial state of the plasma model

Changes from the previous set of models:

- Cancellation of weak coupling between I modes no longer required
- Plasma resistance set to 0 for controller design

Reference ctrl scheme: CREATE v2d0



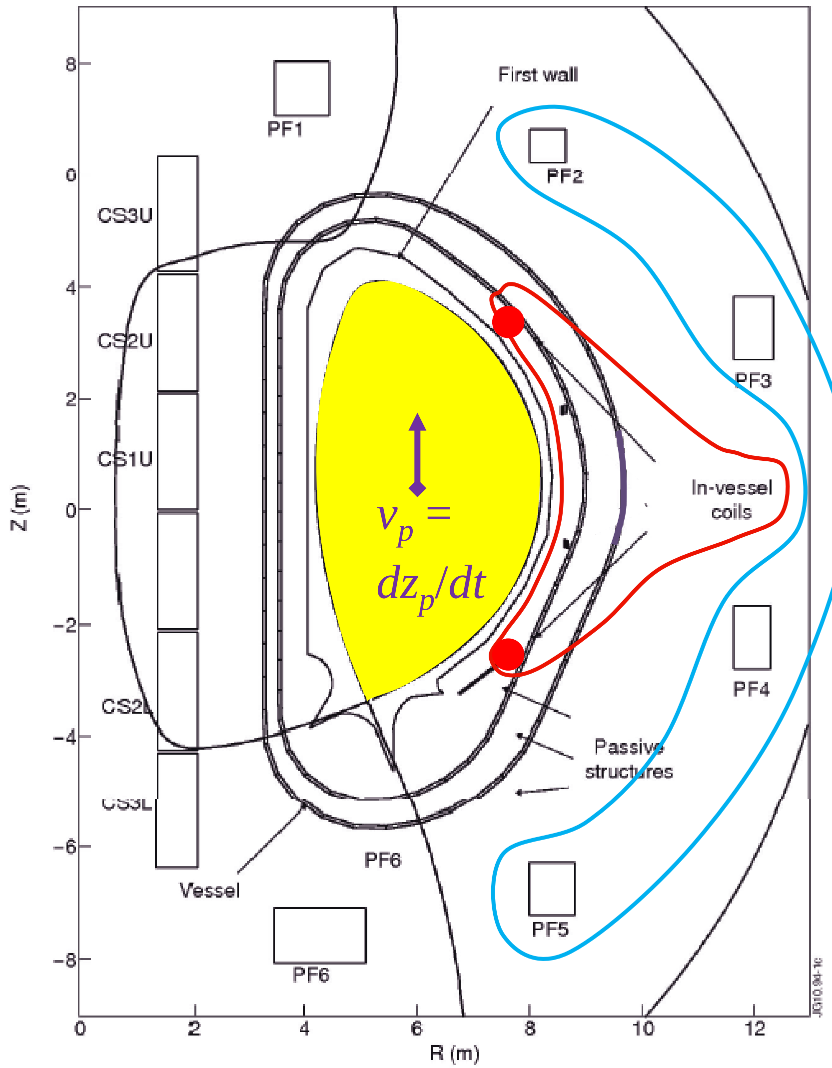
Inner loop: Vertical Stabilisation

Actuators:

- In-vessel coils (Ic) VS3
  $u_1 = u_{ic}$
- Superconductive (Sc) circuit VS1 (PF2-5) $u_2 = u_{VS1}$

Controlled outputs:

- Plasma vertical velocity
  $y_2 = v_p$
- Ic coils current $y_1 = x_{ic}$
  thermal constraint



Outer loop: Plasma Current and Shape Control

Actuators:

- 11 main power supply voltages $\mathbf{V}_{\mathrm{PF}}$

Controlled outputs:

- 11 superconductive coil currents $\mathbf{I}_{\mathrm{PF}}$
- Plasma current I_{p}
- 29 geometrical descriptors $\mathbf{g}$ (2 strike points and 27 gaps)

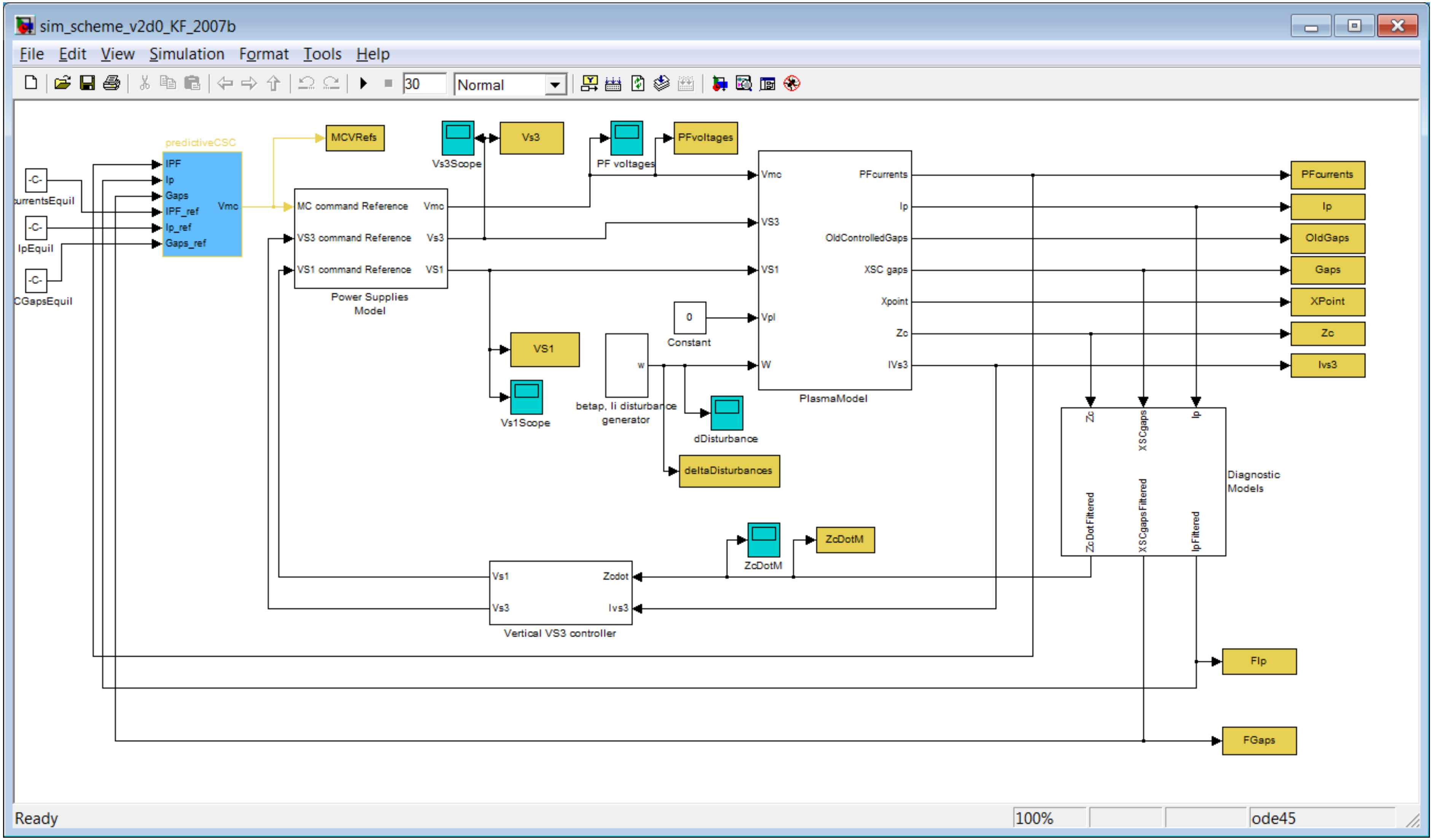
MPC controller for PCSC:

Block predictiveCSC, similar to LQG control

- State estimation using a Kalman Filter
- MPC controller (MPT toolbox)

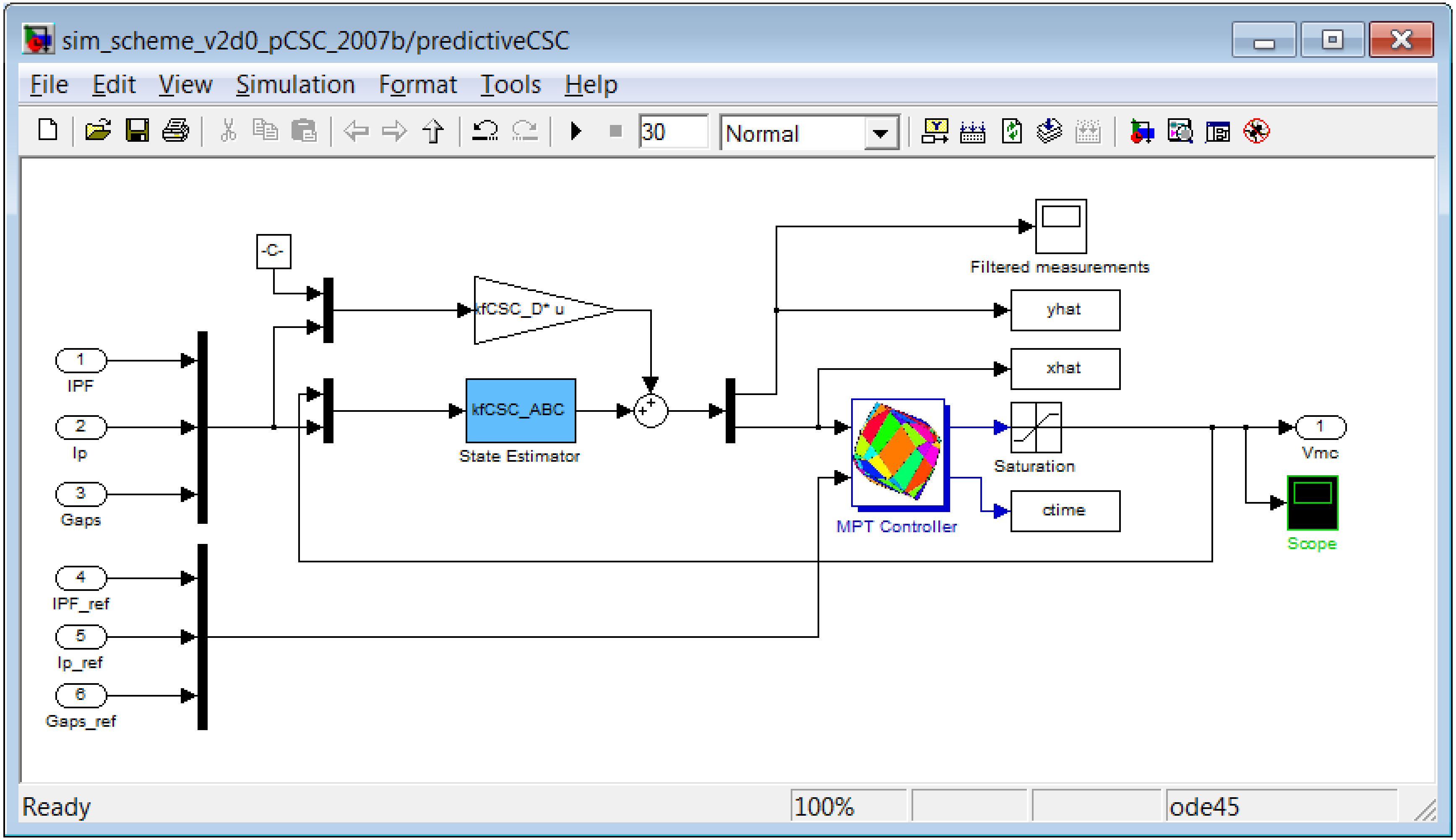
Scheme modified to absolute signals rather than deviations from the operating point, for the sake of constraints handling

Ctrl scheme with MPC PCSC



MPC PCSC – predictiveCSC block

- State estimation using a Kalman Filter
- MPC controller (MPT toolbox)



Model(-based) Predictive Control

- A control methodology in which future control actions are determined by optimisation of a performance criterion defined over a future horizon in which control signals are predicted using a dynamic process model

- It is related to Linear Quadratic optimal control (LQG), they blend in Constrained LQ optimal control

- It may handle constraints on process signals, over a finite horizon

- System $\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k), \;\; \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k)$

- Cost function $J = \sum_{j=0}^{N-1} (\mathbf{x}_{k+j|k}^T \mathbf{Q}_x \mathbf{x}_{k+j|k} + \mathbf{u}_{k+j|k}^T \mathbf{R}_u \mathbf{u}_{k+j|k}) + \mathbf{x}_{k+N|k}^T \mathbf{Q}_{xN} \mathbf{x}_{k+N|k}$

- subject to constraints $\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}, \quad \mathbf{x}_{\min} \le \mathbf{x} \le \mathbf{x}_{\max}$

- Receding-horizon implementation

MPC Implementation

- Solved using a Quadratic Programming solver in each step
  QP: min 0.5 z'Hz + h'z subject to Gz ≤ g, Fz = f
- Write down the sequence of predictions over the horizon, form the cost, build the QP matrices
- May be done "manually"
- Matlab MPC Toolbox: configure via menus
  simple and flexible, if everything you need is supported
- Equation parser to build the QP from a problem description
  YALMIP + modified Multi-Parametric Toolbox (or CVX...)

CSC: Model Predictive Control

MPC is a model-based control method

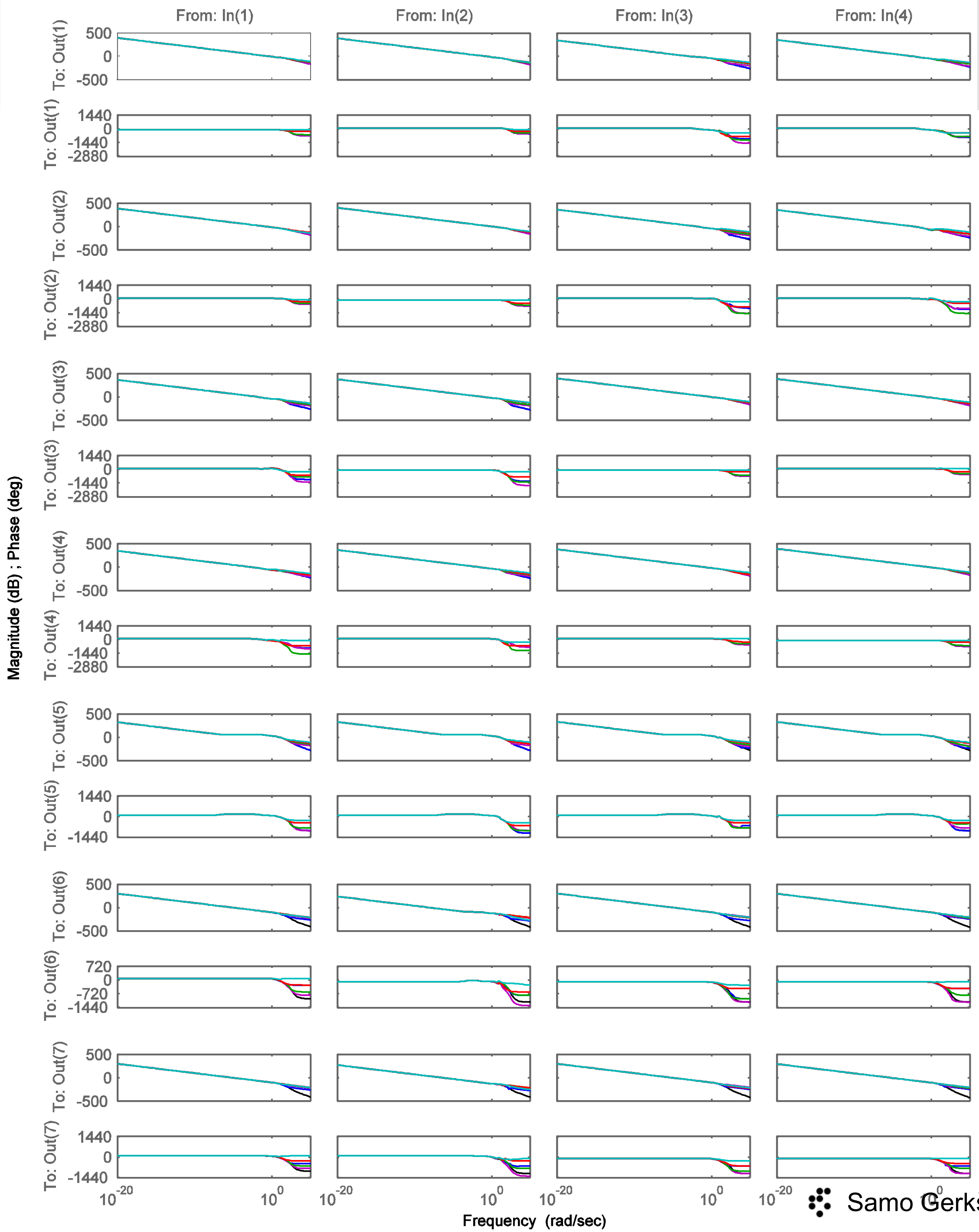
Nominal model t090 preprocessing:

- Extract a state-space model with required inputs and outputs for simulation
- For controller design, set plasma resistance to zero
  - (to avoid issues with model reduction; affects low frequencies only)
- Append simplified power-supply and sensor dynamics
- Compute dynamics with VS feedback (open-loop system for CSC)
- Extract subsystem $\mathbf{u}_{\mathrm{CSC}} = \mathbf{V}_{\mathrm{PF}}$ to $\mathbf{y}_{\mathrm{CSC}} = [\mathbf{I}_{\mathrm{PF}}\ I_{\mathrm{p}}\ \mathbf{g}]^{\mathrm{T}}$
- Remove numerical artefacts at low frequencies using `stabsep`
- Model reduction (200 to 60 states, `balred`, SVD-based)
- Conversion to discrete-time (T_s = 0.1 s, ZOH)
- ...Base model $\{\mathbf{A}_{\mathrm{CSC}}, \mathbf{B}_{\mathrm{CSC}}, \mathbf{C}_{\mathrm{CSC}}, 0\}$

Control of $\mathbf{g}$ and I_{p} with integral action and set-point tracking

- Integral action: disturbance-augmentation, integrators at outputs $\mathbf{g}$, I_{p}
- Set-point tracking: velocity-tracking-augmentation
  to prevent offset when the control signal is non-zero at the steady state,
  $\Delta\mathbf{u}$ becomes the input of the augmented system

Model reduction



Bode diagram

subsystem
from inputs
1, 2, 10, 11
to outputs
1, 2, 10, 11, 12, 13, 14

Unreduced model (black)

Reduced-order models:

- 129 states (blue),
- 80 states (magenta),
- 60 states (green),
- 40 states (red),
- 20 states (cyan)

Sample time Ts

- Discrete-time controllers: Ts must be chosen
- A relatively wide range of useful Ts, rules of thumb...
- MPC: the problem of computational demand, Ts > Tcomp
  Predictive horizon N, in terms of time N*Ts
    should cover the system settling time
  Even with inf-horizon MPC, N*Ts affect the ability to respond to constraints
- Small N (e.g. 10) preferred computationally, common in theoretical papers
  Ts = 1 s ... stable control but sluggish response to disturbances
- Response to disturbances no longer impaired at Ts = 0.1 ... N around 30

Kalman Filter tuning

Covariance matrices $\mathbf{Q}_K$ and $\mathbf{R}_K$ for the disturbance-augmented system

Theory: estimate noise covariances E{$\mathbf{w}_a\mathbf{w}_a^T$} and E{$\mathbf{v}\mathbf{v}^T$}

...infeasible with a non-existing system

...result may be optimal w.r.t. system and l_2 cost function, but not practically

Practice: "observer approach",

diagonal elements of $\mathbf{Q}_K$ and $\mathbf{R}_K$ considered tuning parameters

Grouping of elements to reduce the number of tuning parameters

For instance: $\mathbf{Z} = \mathrm{diag}(\mathbf{B}^T\mathbf{B})$

$$\mathbf{Q}_K = \begin{bmatrix} k_3\mathbf{C}\mathbf{C}^T & & & \\ & k_4\mathbf{I}_{11} & & \\ & & k_5\mathbf{Z}_{12} & \\ & & & k_6\mathbf{Z}_{13:41} \end{bmatrix}, \qquad \mathbf{R}_K = \begin{bmatrix} \mathbf{Z}_{1:11} & & \\ & k_1\mathbf{Z}_{12} & \\ & & k_2\mathbf{Z}_{13:41} \end{bmatrix}$$

base-model states ($k_3\mathbf{C}\mathbf{C}^T$)   disturbance I states (k_4, k_5, k_6 blocks)

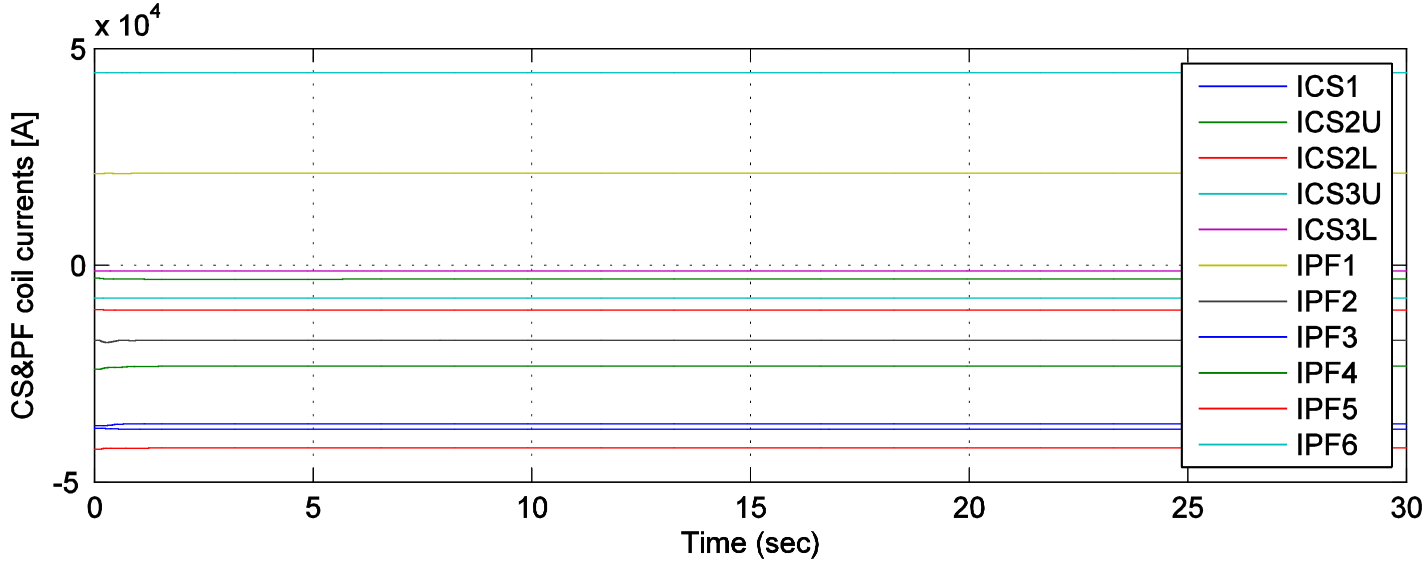
Initial tuning: CSC in open loop (VS only)

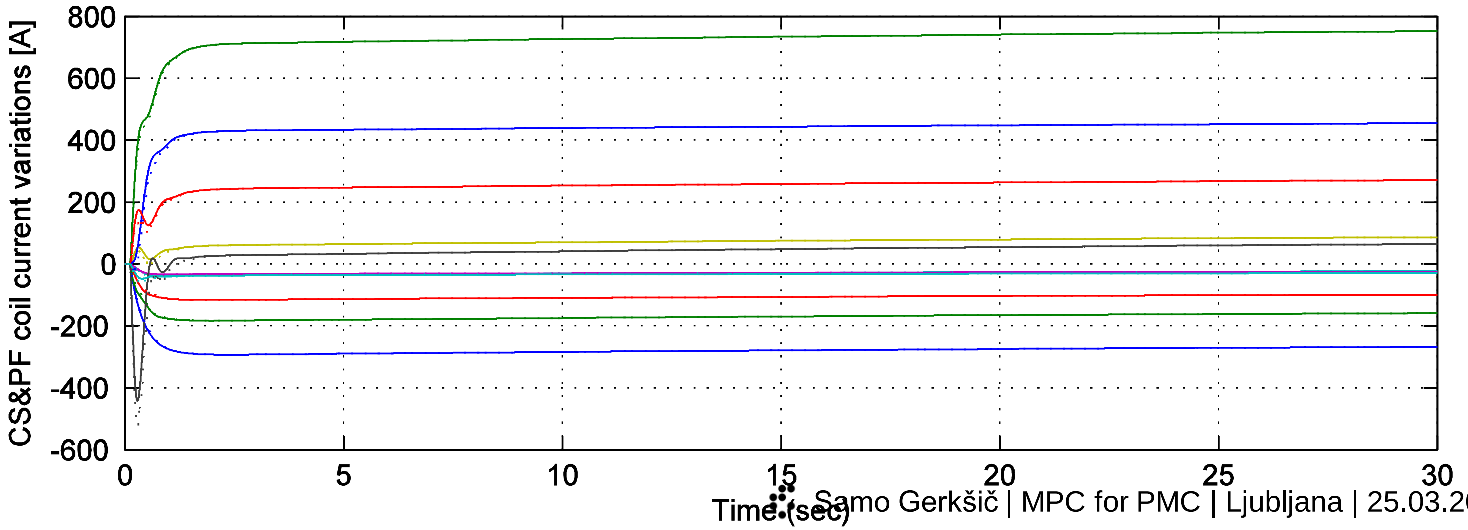
Final tuning: CSC in closed loop, interaction with the controller

KF state estimation, CSC open-loop

Minor disruption simulation: SC&PF coil currents (top: absolute, bottom: displacements)

Estimates: dotted lines, bottom only

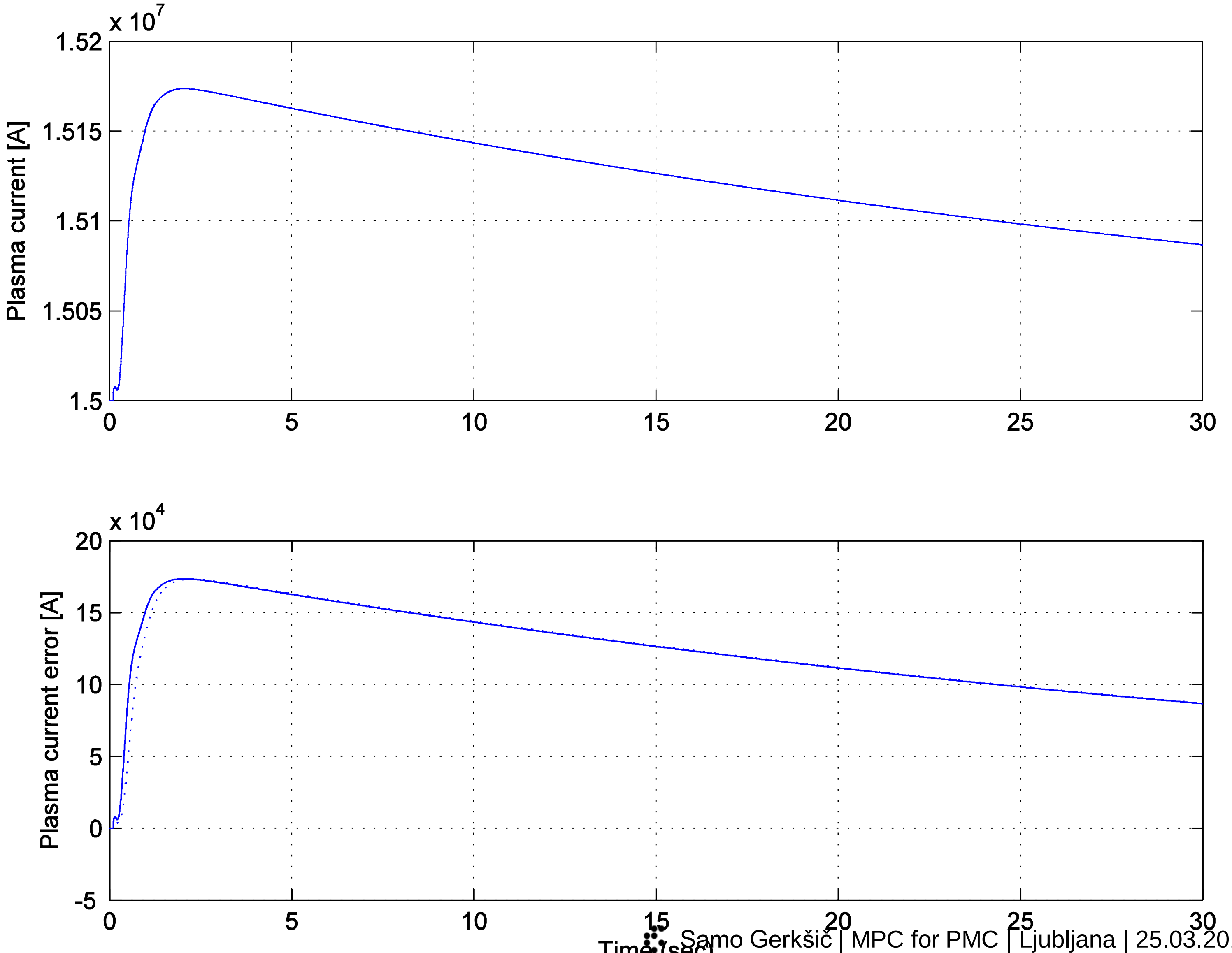




KF state estimation, CSC open-loop

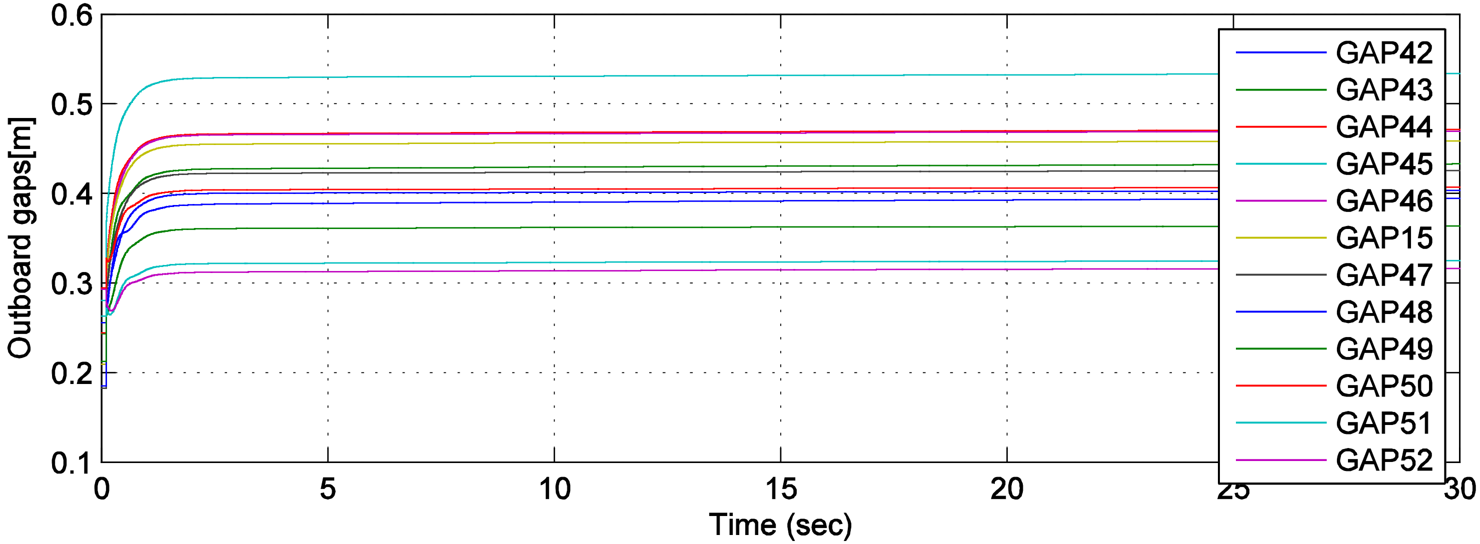
Minor disruption simulation: Plasma current (top: absolute, bottom: displacement)

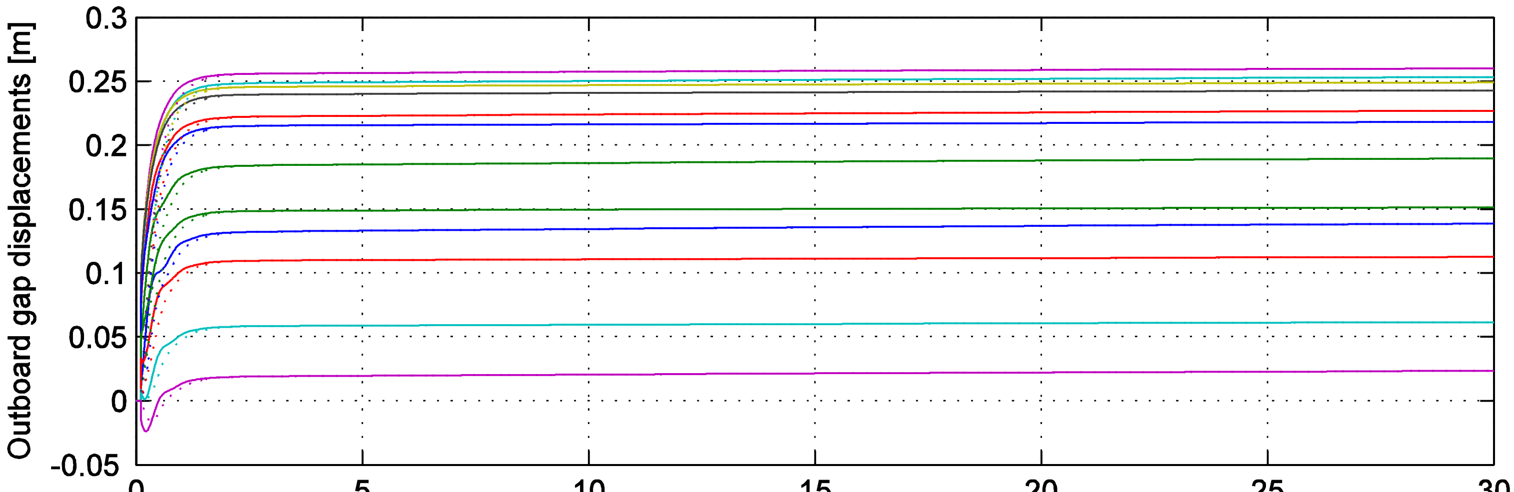
Estimates: dotted lines, bottom only



KF state estimation, CSC open-loop

Minor disruption simulation: Outboard gaps (top: absolute, bottom: displacements)
Estimates: dotted lines, bottom only





MPC PCSC variants

- Initial CSC prototype
  Available at project start, used as benchmark for QP algorithms
  Differences: regulation of deviation signals, to 0 without set-point tracking,
  g with 6 elements (4 gaps and 2 strike-points); different models
- MPC CSC with full output vector
- MPC CSC with reduced output vector: manual selection
- MPC CSC with reduced output vector: manual selection and averaging
- MPC CSC with reduced output vector: SVD of C matrix
- MPC CSC with reduced output vector: static SVD

MPC CSC with full output vector

- Manipulated variable dimension: 11
- Controlled Variable dimension: 11+1+29 = 41
- Control without offset in steady state is not possible
  (degrees of freedom lacking)
- Difficult to tune control trade-offs
- Computationally inconvenient (large dimension)
  ...MPT toolbox fails

MPC CSC with reduced output vector: manual selection

Control only selected gaps $\mathbf{g}_{\mathrm{sel}}$ instead of all gaps $\mathbf{g}$

Introduce the output selection matrix $\mathbf{M}_{\mathrm{sel}}$ (containing mostly zeros, and n_g elements equal to 1, one in each row)

$$\mathbf{g}_{\mathrm{sel}} = \mathbf{M}_{\mathrm{sel}}\mathbf{g}$$

- Manipulated Variable dimension: 11
- Controlled Variable dimension: 11+1+6 = 18
- Control without offset in steady state is possible for the selected gaps (other gaps have offset, are not estimated & controlled)
- With n_g < 10, DoF remaining for response to constraints
- Similar to the prototype MPC CSC in performance and computational complexity
- Implementation:
  $\mathbf{C}_{\mathrm{CSC}}$ is replaced with a reduced matrix $\mathbf{C}_{\mathrm{CSCsel}}$

MPC CSC with reduced output vector: manual selection and averaging

Individual selected gaps may be replaced with weighted sums (averages) of neighbouring gaps $\mathbf{g}$

For instance, $\mathbf{g}_{\text{sel}}$ and $\mathbf{M}_{\text{sel}}$ from the list:

```
gsel{1} = 1:12;   % inboard gaps
gsel{2} = 13:15;  % top gaps
gsel{3} = 16:19;  % top outboard gaps
gsel{4} = 20:27;  % bottom outboard gaps
gsel{5} = 28;     % strike point GAP25
gsel{6} = 29;     % strike point GAP21
```

- Control without offset in steady state is possible for the selected gaps or their weighted sums
  (other gaps, incl. individual gaps in sums, have offset)
- Computational complexity as previous;
  but control considers more gaps
- Implementation:
  $\mathbf{C}_{\text{CSC}}$ is replaced with a reduced matrix $\mathbf{C}_{\text{CSCsel}}$ (averaging of rows)

MPC CSC with reduced output vector: SVD of C matrix

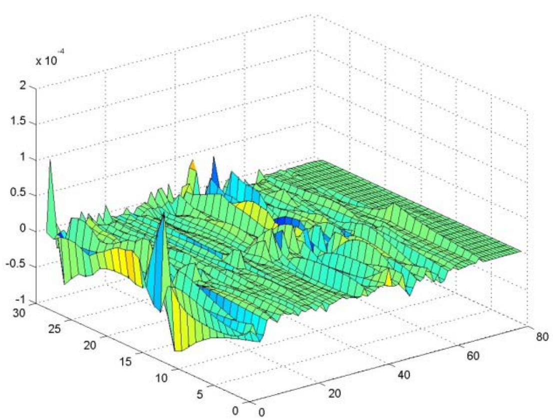
Apply SVD to $\mathbf{C_g}$ (the part of the output matrix $\mathbf{C}_{CSC}$ producing the geometrical descriptors **g**): $\mathbf{C_g} = \mathbf{U}_0\mathbf{S}_0\mathbf{V}_0^T$

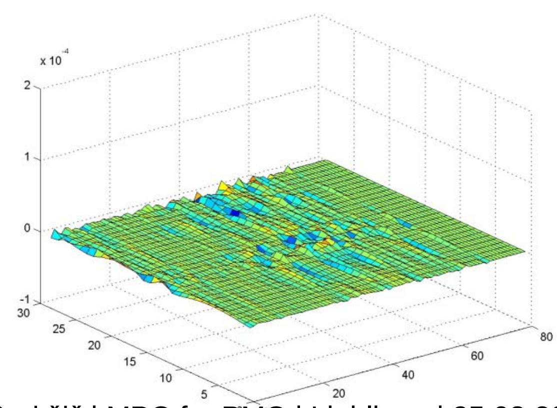
Truncated SVD using first n_g singular values: $\mathbf{C_{g1}} = \mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^T$

Artificial output $\mathbf{g}_{SVD}$, dim n_g: $\mathbf{g} = \mathbf{U}_1\mathbf{S}_1\,\mathbf{g}_{SVD}$, $\mathbf{g}_{SVD} = \mathbf{V}_1^T\,\mathbf{x}$

- Control without offset in steady state is possible for $\mathbf{g}_{SVD}$ (gaps have offset)
- Smaller n_g: more offset
- Problem: too much offset with reasonable n_g!

Surface plots of elements of $\mathbf{C_{g1}}$(left), and the difference ($\mathbf{C_g}$ - $\mathbf{C_{g1}}$) (right), $n_g = 6$





MPC CSC with reduced output vector: static SVD

Apply SVD in a "static" manner, to the sub-matrix of **C** from $\mathbf{I}_{PF}$ to $\mathbf{g}$:

$\mathbf{C}_s$ = LinearModel.C(GapIndexout, PFindexShape);

$\mathbf{C_s} = \mathbf{U}_0\mathbf{S}_0\mathbf{V}_0^T$

Truncated SVD using first n_g singular values: $\mathbf{C_{s1}} = \mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^T$

Artificial output $\mathbf{g}_{\mathrm{SVD}}$, dim n_g: $\mathbf{g} = \mathbf{U}_1\,\mathbf{g}_{\mathrm{SVD}}$, $\mathbf{g}_{\mathrm{SVD}} = (\mathbf{U}_1^T\mathbf{U}_1)^{-1}\mathbf{U}_1\ \mathbf{g}$

Modified part of C matrix for gaps: $\mathbf{C_{g1}} = (\mathbf{U}_1^T\mathbf{U}_1)^{-1}\mathbf{U}_1\ \mathbf{C_g}$

in fact, weighted averaging of rows, weights from SVD: $\mathbf{M}_{\mathrm{sel}} = (\mathbf{U}_1^T\mathbf{U}_1)^{-1}\mathbf{U}_1)$

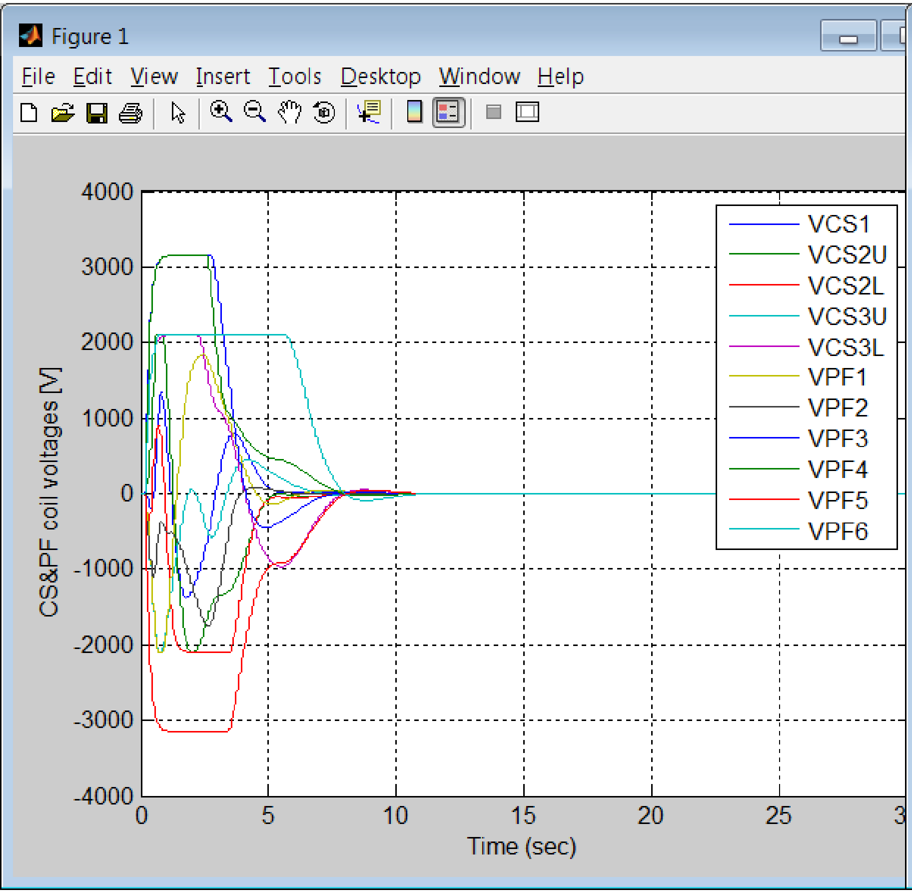
- Control without offset in steady state is possible for $\mathbf{g}_{\mathrm{SVD}}$ (gaps have offset)
- Smaller n_g: more offset, but less control effort ($\mathbf{I}_{PF}$) in the steady state, control looks reasonable with n_g = 6..9

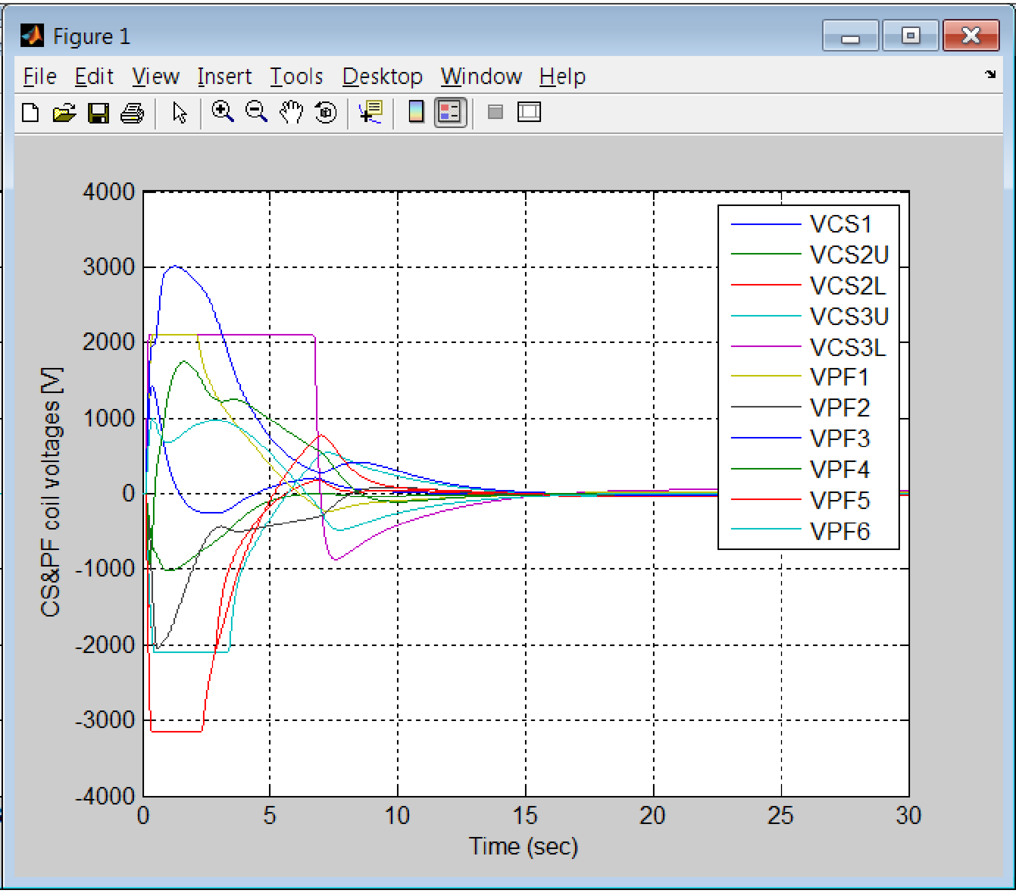
...sample simulation result with provisional tuning:

MPC CSC static SVD n_g=9

Minor disruption simulation: SC&PF coil voltages

Left: MPC CSC, right: CREATE v2d0

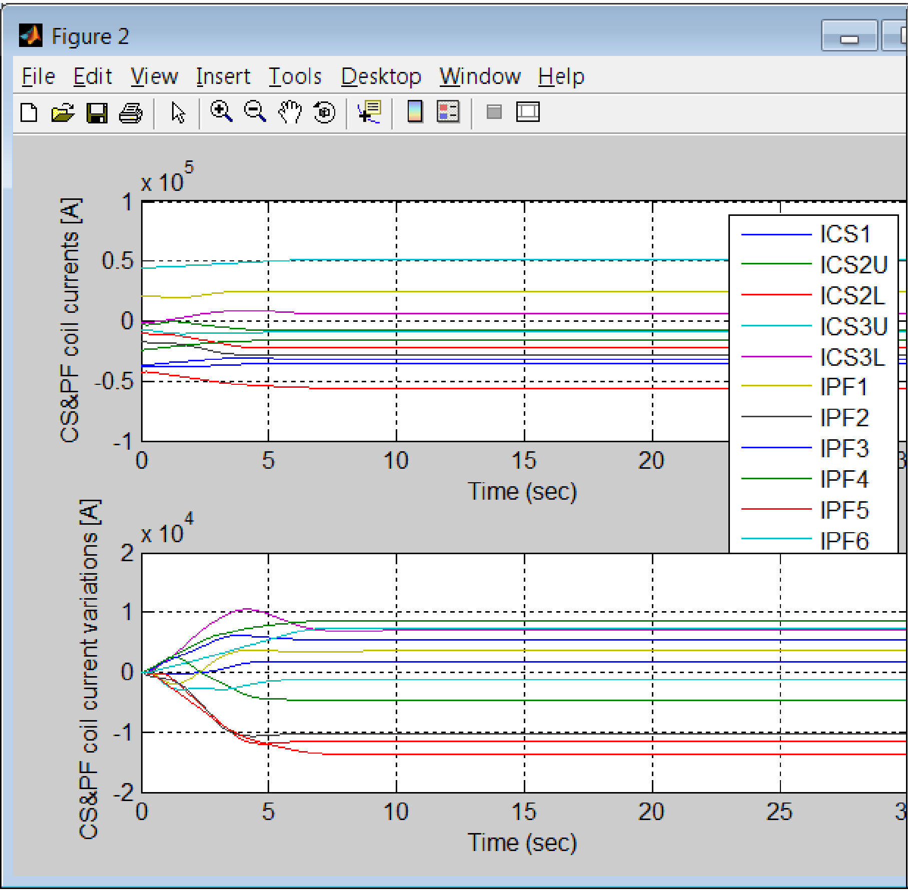


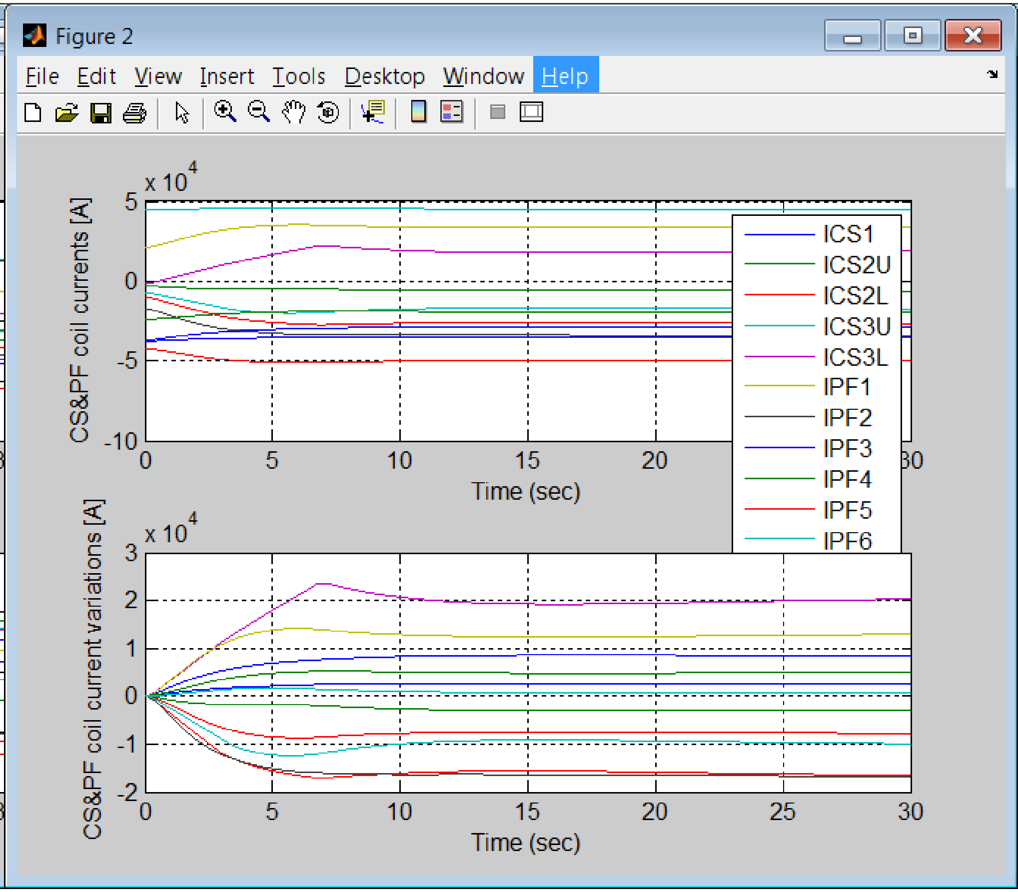


MPC CSC static SVD $n_g=9$

Minor disruption simulation: SC&PF coil currents (top: absolute, bottom: displacements)

Left: MPC CSC, right: CREATE v2d0

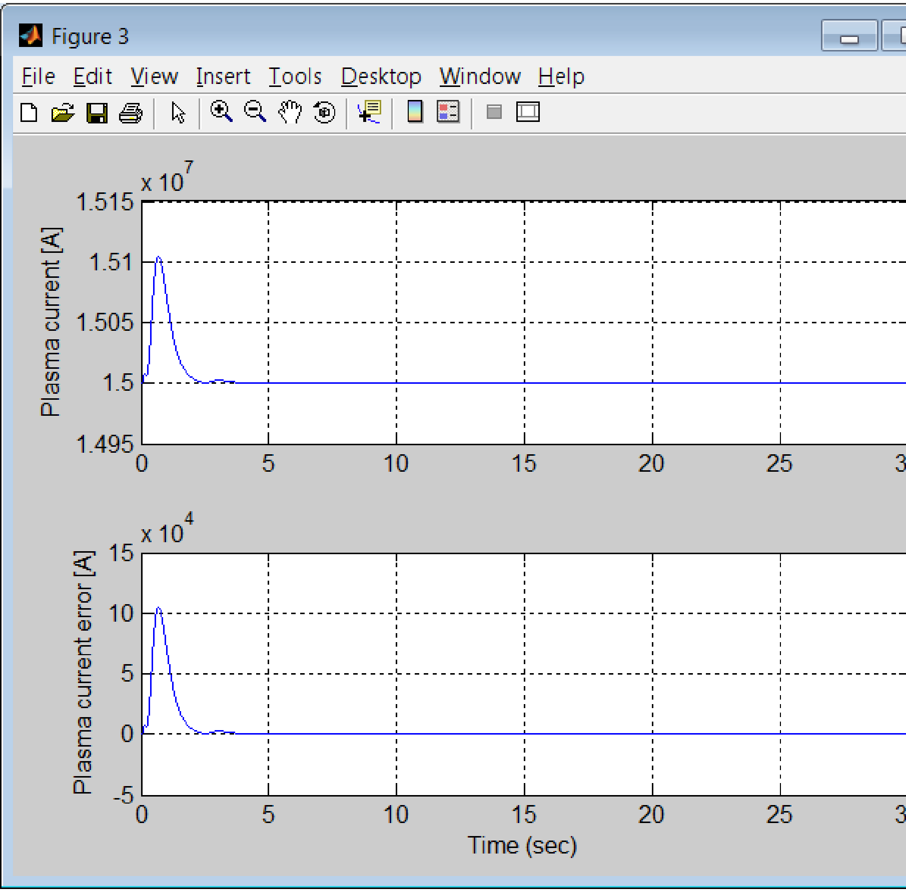


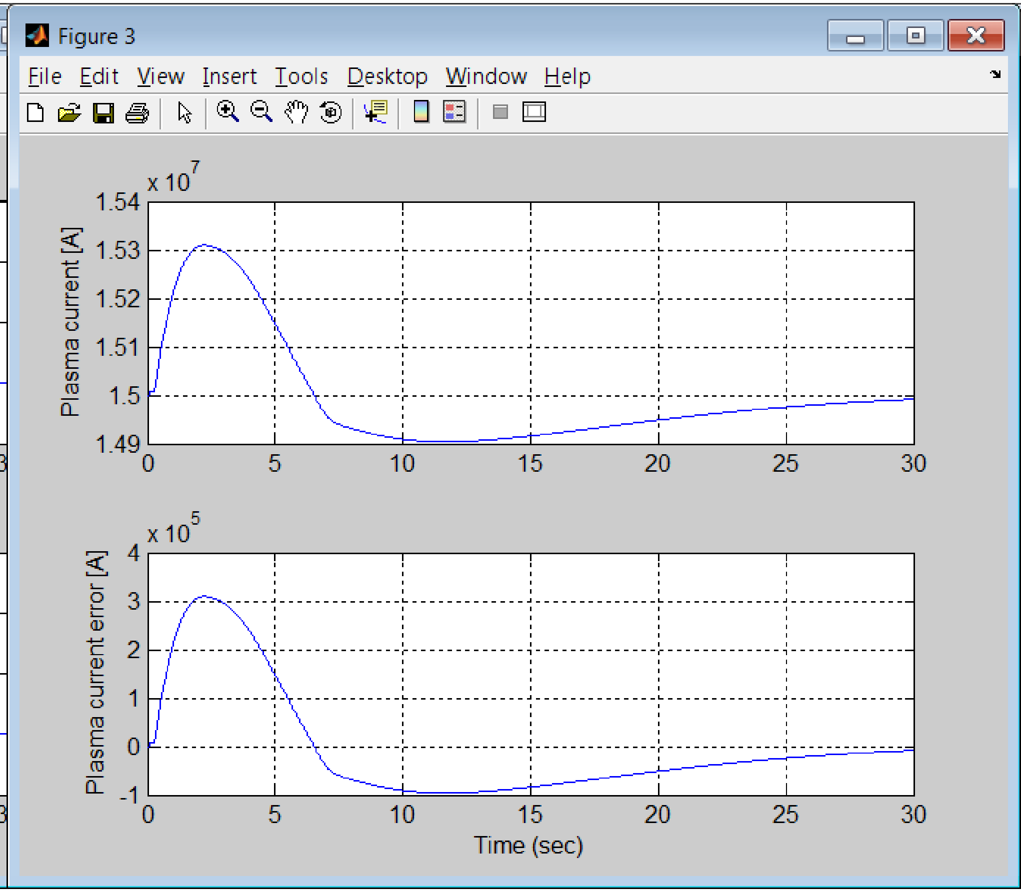


MPC CSC static SVD n_g=9

Minor disruption simulation: Plasma current (top: absolute, bottom: displacements)

Left: MPC CSC, right: CREATE v2d0

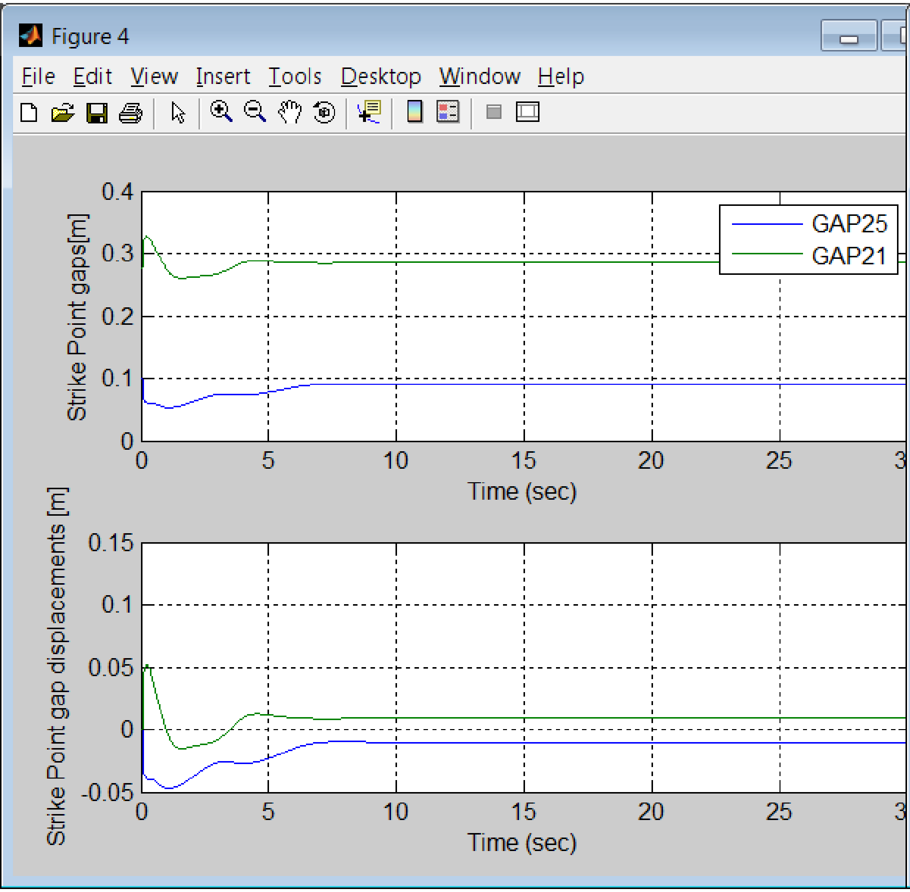


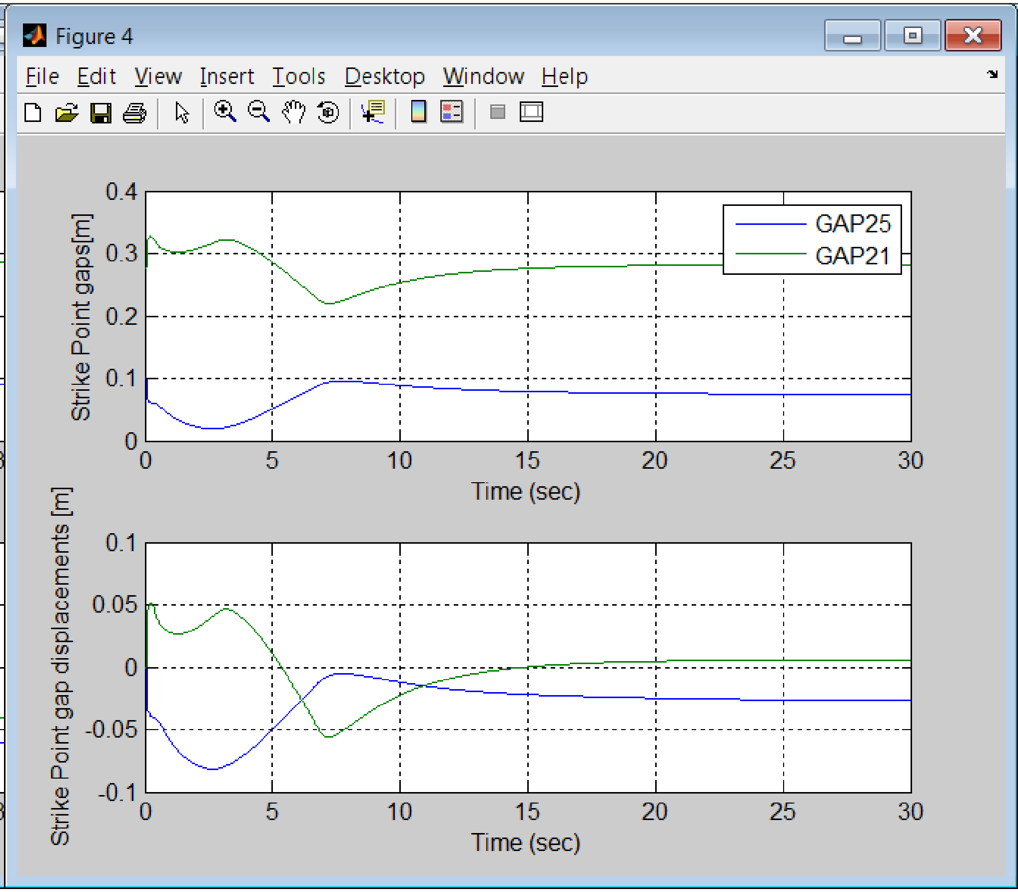


MPC CSC **static SVD** n_g=9

Minor disruption simulation: Strike points (top: absolute, bottom: displacements)

Left: MPC CSC, right: CREATE v2d0

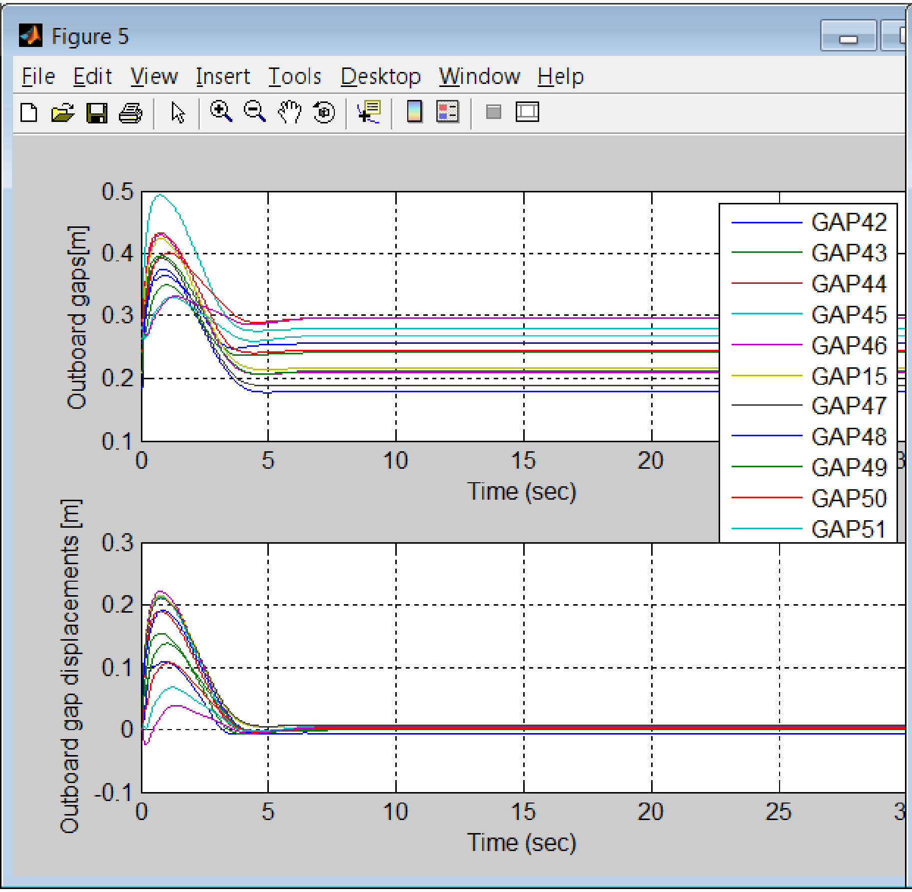


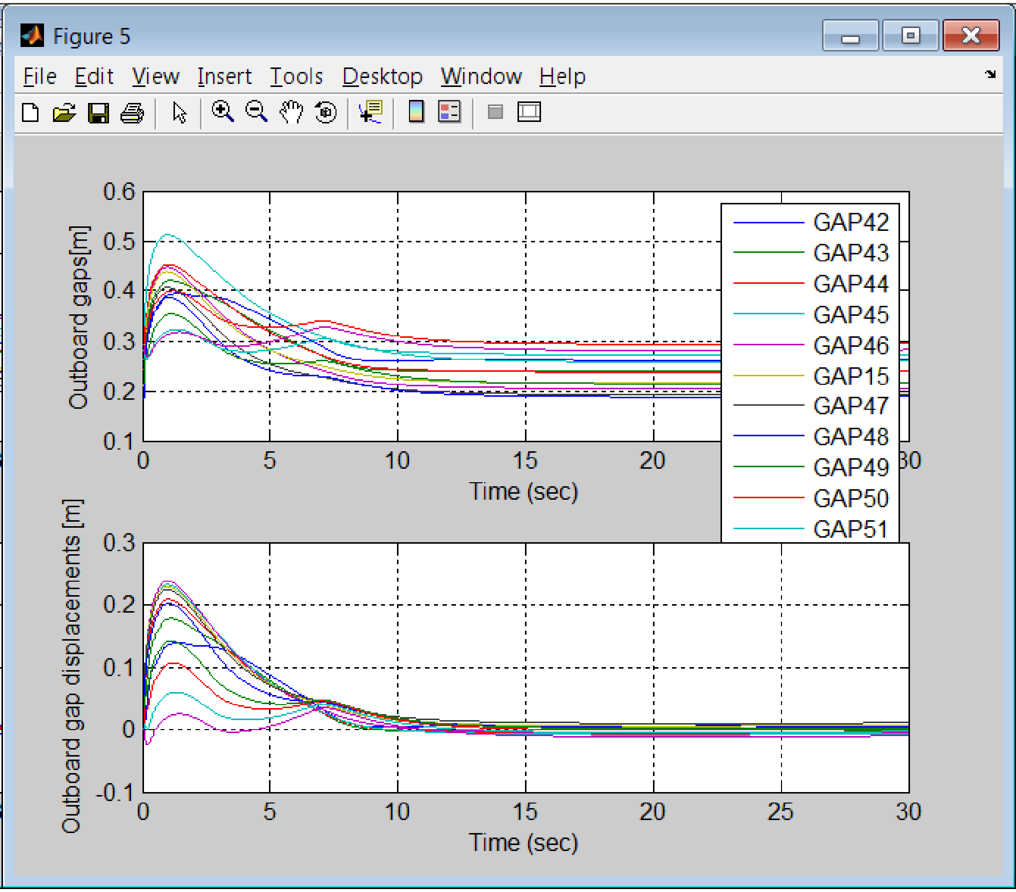


MPC CSC static SVD n_g=9

Minor disruption simulation: Outboard gaps (top: absolute, bottom: displacements)

Left: MPC CSC, right: CREATE v2d0

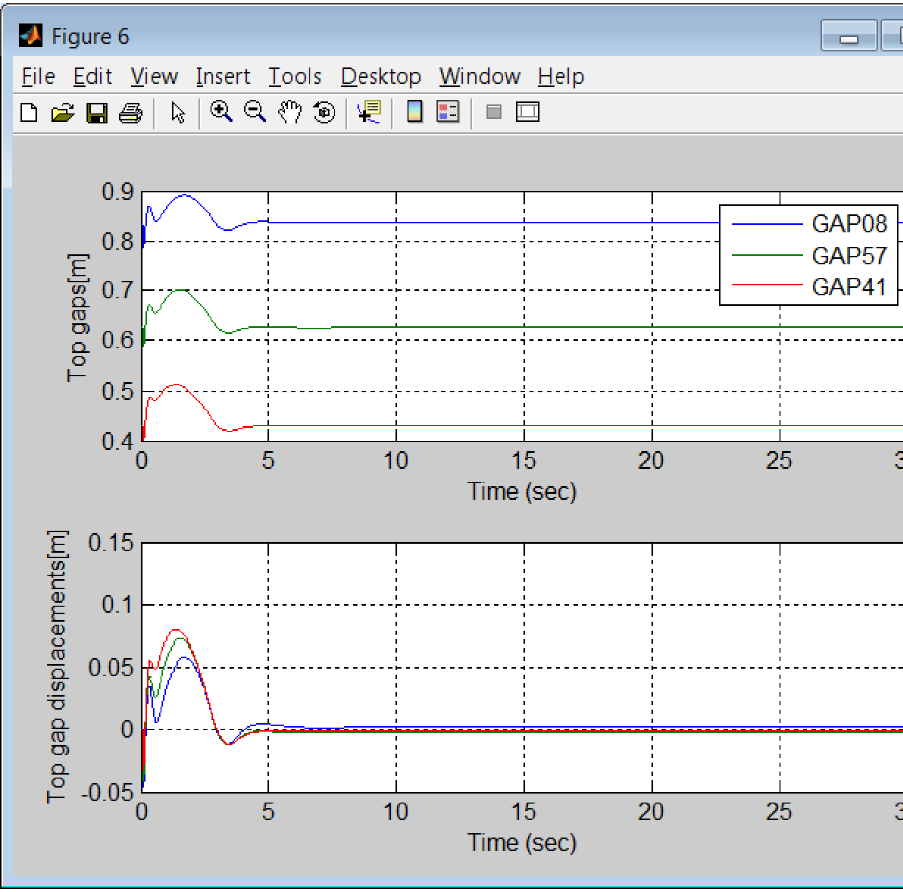


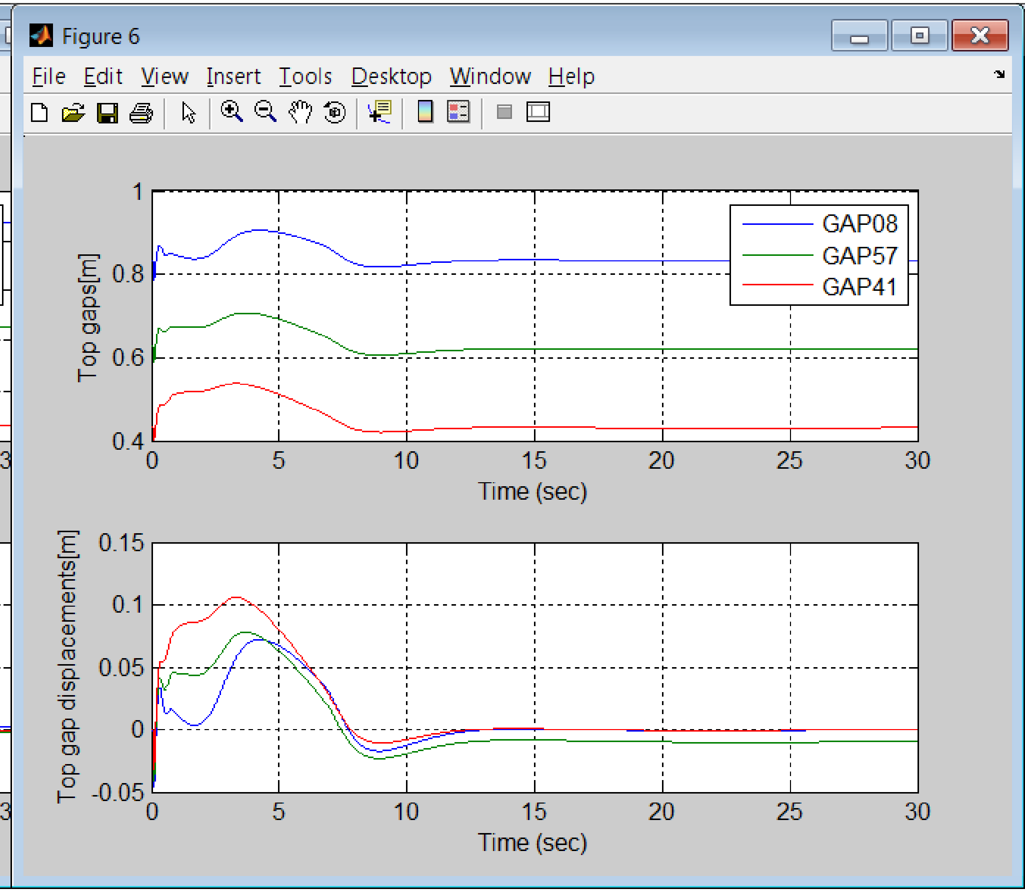


MPC CSC static SVD n_g=9

Minor disruption simulation: Top gaps (top: absolute, bottom: displacements)

Left: MPC CSC, right: CREATE v2d0

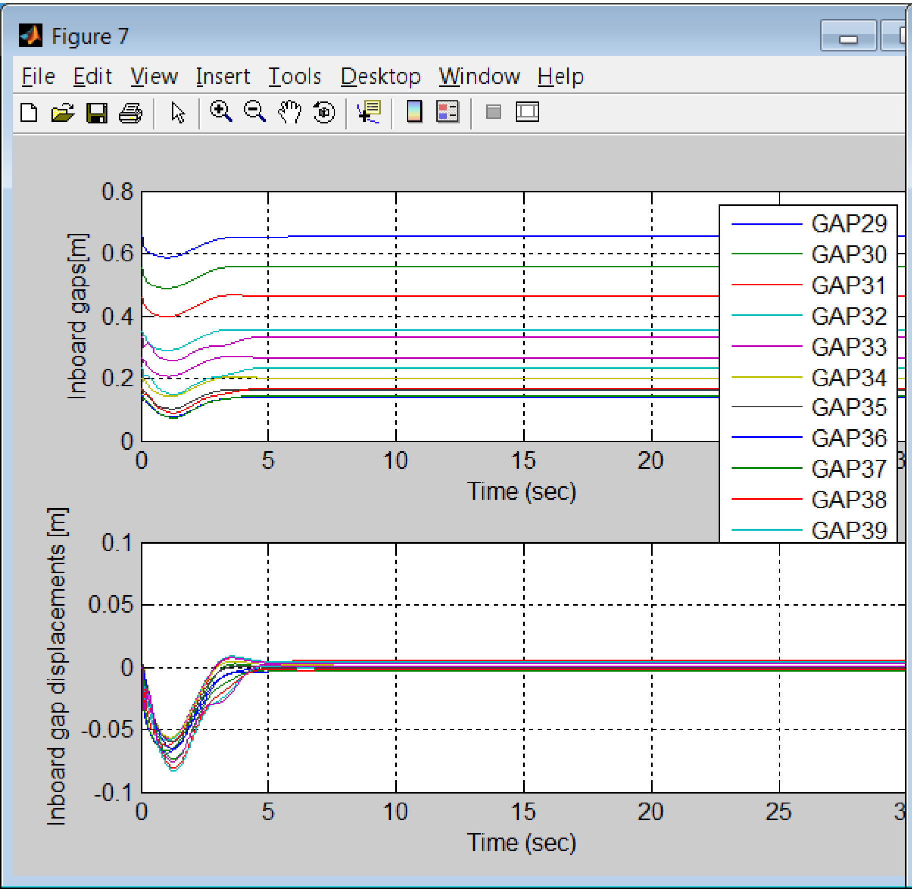


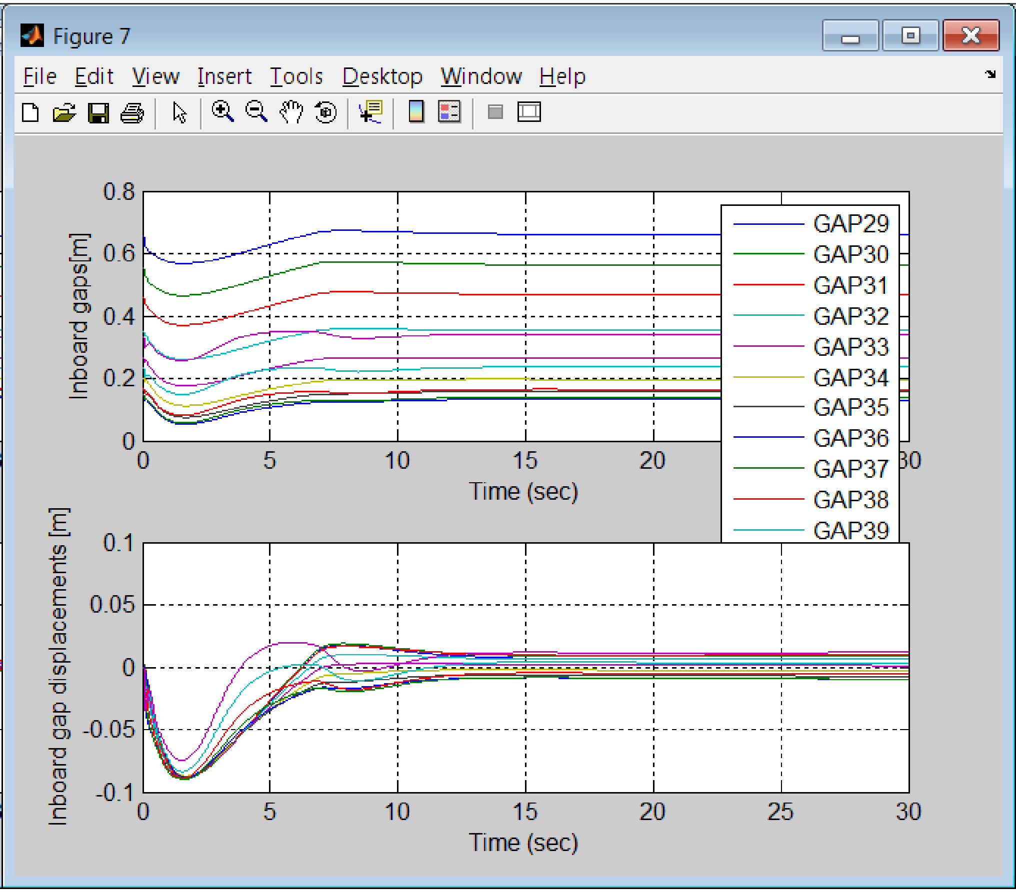


MPC CSC static SVD n_g=9

Minor disruption simulation: Inboard gaps (top: absolute, bottom: displacements)

Left: MPC CSC, right: CREATE v2d0





MPC CSC **static SVD** n_g=9

Minor disruption simulation evaluation:

MPC PCSC ng=9:

Maximum CS&PF Power during simulation:
2.6509e+008

Current limits and Maximum abs of the currents during the simulation:

1.0e+004 *

4.5000	3.7878
4.5000	0.7693
4.5000	2.2156
4.5000	1.0449
4.5000	0.9167
4.8000	2.4976
5.5000	2.8046
5.5000	3.7009
5.5000	2.4012
5.5000	5.6161
4.8000	5.1970

Minimum plasma-wall gap during simulation:
'GAP37'

ggmin = 0.0733

Minor disruption simulation evaluation:

CREATE v2d0:

Maximum CS&PF Power during simulation:
2.3949e+008

Current limits and Maximum abs of the currents during the simulation

1.0e+004 *

4.5000	3.7620
4.5000	0.6139
4.5000	2.7348
4.5000	1.9961
4.5000	2.2244
4.8000	3.5334
5.5000	3.4100
5.5000	3.7009
5.5000	2.4012
5.5000	5.1131
4.8000	4.6272

Minimum plasma-wall gap during simulation
'GAP36'

ggmin = 0.0530

Conclusions

Roughly reasonable performace is achieved with the "static SVD" scheme

Tuning is provisional only; the controller is not finalized yet

Further work:

- Finalization of the SVD approach
- Target Calculator scheme
- Tuning
- Performance with constraints
- Performance evaluation with a set of linear models
- Performance evaluation with the nonlinear model
- Fast QP implementation