CREATE ITER RWM control system

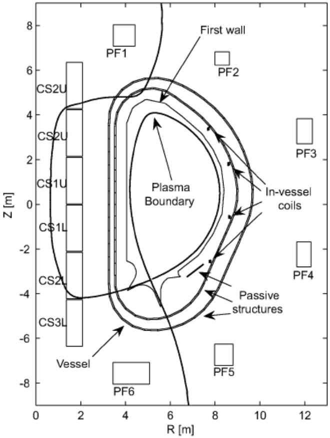
Marco Ariola

on the behalf of the CREATE team

Progress Meeting – Fast MPC for Magnetic Plasma Control

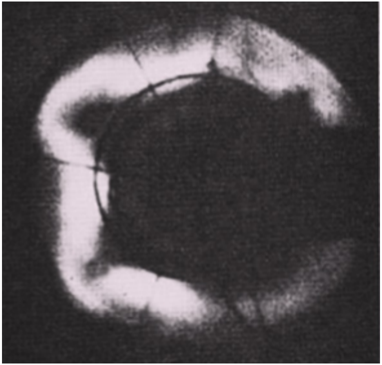
Outline

Resistive Wall Modes - 1



- Tokamak control systems have to deal with different kinds of instabilities related to the presence of a resistive wall that surrounds the plasma
- The main instability is due to an axisymmetric ($n = 0$) mode, the so-called axisymmetric Vertical Displacement Event, which occurs whenever a plasma with a vertical elongated poloidal cross-section is operated

Resistive Wall Modes - 2



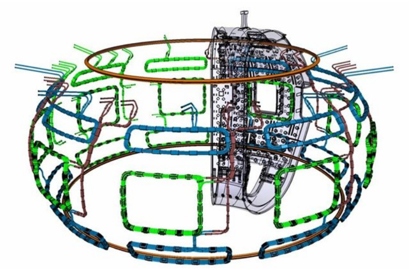
- Another important plasma instability is the one called kink instability, which is the main non-axisymmetric ($n = 1$) mode
- The kink instability arises when the *plasma pressure* exceeds a certain threshold $\rightarrow$ it is similar to a garden hose kinking when it is suddenly pressurized

Control of RWMs

- Elongated plasmas enable to increase the energy confinement time, which is an essential criterion for realizing sustained fusion, but they are vertically unstable
  - The use of an active feedback system, usually called **vertical stabilization system** is required
- Modern tokamak devices operate at high plasma pressure, hence a kink instability is most likely to occur
  - a **control system to stabilize also the** $n = 1$ **mode** becomes necessary

Control coils

The 27 non-axisymmetric coils so-called ELM coils are used to stabilize the $n = 1$ mode. The ELM coils are three for each of the nine sectors; the sectors are equally-spaced and located at the toroidal angles $\eta_i = 40^o \cdot (i - 1)$, with $i = 1, \ldots, 9$



Voltages applied to the ELM coils - 1

- In ITER the stabilization of the resistive wall modes relies on a set of 27 coils
- Each one of these coils is fed by an independent power supply that, from a control design point of view can be modelled as a first order with delay transfer function given by

$$H_{PS} = \frac{1}{7.5 \times 10^{-3} s + 1} e^{-2.5 \times 10^{-3} s}$$

- Moreover constraints on the maximum voltage $V_{\max}$ and on maximum current $I_{\max}$ are given. Hereafter we assume

$$V_{\max} = 144 V\,, \qquad I_{\max} = 16 kA$$

- The resistive wall mode stabilizing coils are placed along three toroidal sectors, the upper, the equatorial, and the lower sectors.
- Each toroidal sector contains nine coils, covering each one an angle of 40^{o}
- We denote by η_i, $i = 1, 2, \ldots, 9$ the toroidal angle at the center of each coil

Voltages applied to the ELM coils - 2

- Since we want to control magnetic perturbations with a sinusoidal dependence (with respect to the toroidal angle φ) without exciting higher number toroidal harmonic, we assume that the voltages have a sinusoidal behavior
- Assume that $u_i \in \mathbb{R}^{9\times 1}$, $i \in \{1,2,3\}$ are the voltages applied to the ELM coils in the upper, center, and lower region
- These voltages are decomposed in the following way

$$u_i = \Theta \cdot \tilde{u}_i \,, \quad i = 1,2,3 \,, \tag{1}$$

where

$$\Theta = \begin{pmatrix} \cos\eta_1 & \sin\eta_1 \\ \cos\eta_2 & \sin\eta_2 \\ \cdots & \cdots \\ \cos\eta_9 & \sin\eta_9 \end{pmatrix} \in \mathbb{R}^{9\times 2} \,.$$

Voltages applied to the ELM coils - 3

- In this way the controller will act on the cosine and sine components of the voltage distribution along the toroidal angle, and will provide a quasi-sinusoidal compensating magnetic field.
- The compensating magnetic field is not perfectly sinusoidal because the coil currents along the 40º covered by each coil is constant and do not vary according to a sinusoidal pattern.
- Letting $u = [u_1^T \ u_2^T \ u_3^T]^T \in \mathbb{R}^{27}$, and $\tilde{u} = [\tilde{u}_1^T \ \tilde{u}_2^T \ u_3^T]^T \in \mathbb{R}^6$, we can write

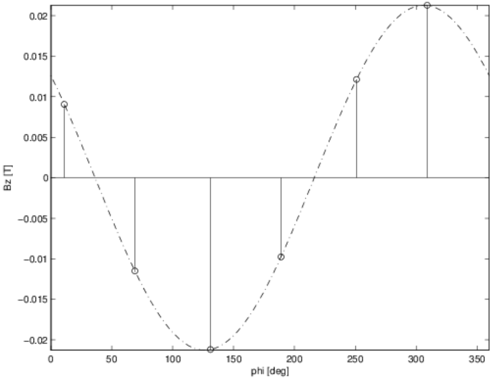
$$u = \begin{pmatrix} \Theta & 0 & 0 \\ 0 & \Theta & 0 \\ 0 & 0 & \Theta \end{pmatrix} \tilde{u} = T_{in}\tilde{u}$$

where $T_{in} \in \mathbb{R}^{27\times 6}$

Controlled variables for RWM stabilization - 1

- Since the $n = 1$ mode is not axisymmetric it cannot be detected by means of magnetic measurements located in one poloidal section, as it happen when considering the vertical stabilization or the plasma shape control problems
- In ITER the standard measurement set for the stabilization of the resistive wall modes is composed by 6 vertical field sensors located in the outboard side of the machine
- The sensors are located all in the same poloidal position, but at different toroidal angles spaced of about 60^o.
- These sensors allow us to reconstruct the amplitude and phase of the $n = 1$ magnetic field perturbation acting along a circle passing through the point of coordinates (8.928, 0.550)
- This point is at the height of the plasma current centroid for most ITER plasma reference equilibria
- In our approach, stabilization of the $n = 1$ resistive wall modes is equivalent to maintain to zero the amplitude of the magnetic field perturbation as measured by the available set of measurements

Controlled variables for RWM stabilization - 2



- This figure shows the vertical magnetic field measured by the sensors at the initial time, when an initial condition lying on the plane of the eigenvectors corresponding to the unstable eigenvectors is given
- As this figure shows, the dependence with respect to the toroidal angle φ is almost of sinusoidal type

Controlled variables for RWM stabilization - 3

- Denoting with φ_i the toroidal angle at which the sensors are located, the amplitude of the magnetic field to be controlled can be evaluated, starting by the measurements, assuming

$$y_i = y_A \cos\varphi_i + y_B \sin\varphi_i \,, \quad i = 1, 2, \ldots, 6$$

- The cosine and sine coefficients can be estimated using the minimum mean square formula

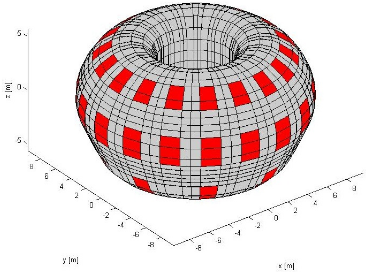
$$\tilde{y} = \begin{pmatrix} y_A \\ y_B \end{pmatrix} = \begin{pmatrix} \cos\varphi_1 & \sin\varphi_1 \\ \cos\varphi_2 & \sin\varphi_2 \\ \vdots & \vdots \\ \cos\varphi_6 & \sin\varphi_6 \end{pmatrix}^{\dagger} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_6 \end{pmatrix} = T_{out} y \,,$$

  where $T_{out} \in \mathbb{R}^{2\times 6}$.
- The amplitude of the magnetic perturbation seen by the sensors can be evaluated as

$$Y = \sqrt{y_A^2 + y_B^2}$$

Plant model - 1

- The ITER tokamak has been discretized with a 3D finite elements mesh, made of 4970 hexahedral elements, giving rise to $N = 4135$ discrete degrees of freedom
- The mesh takes approximately into account the presence of ports and port extensions, using some conducting patches on the vessel with an equivalent resistivity (shown in red)
- The considered plasma equilibrium is a $I_p = 9$ MA configuration, with a normalized $\beta_N = 2.94$ (this parameter quantifies the plasma pressure)



Plant model - 2

For controller design purposes the following linearized model can be considered

$$\dot{x} = Ax + Bu$$
$$y_m = C_m x$$

where

- the state vector x coincides with the set of the edge current displacements
- the input quantities u are the voltages fed to the 27 active coils
- the measured outputs y_m are the magnetic field displacements at given spatial points around the torus

Given the 3D finite elements discretization, the order of the model is about four thousand.

Unstable modes

- The dynamic matrix A has two unstable eigenvalues
- Each of the related eigenvectors corresponds to a specific current pattern inside the three-dimensional structure
- These unstable eigenvalues have the same value (about $27s^{-1}$) and correspond to two $n = 1$ current density patterns, which are identical apart from a shift of $\pi/2$ in the toroidal direction
- Numerical errors in the linearization procedure make these two eigenvalues to have a very small imaginary part

Null controllable region - 1

- The voltage and currents limits of the power supplies constraint the amplitude of the maximum perturbations which can be recovered
- A method to estimate such maximum perturbations is the evaluation of the null controllable region of our system when the power supplies are subject to given maximum voltage constraints.
- *An initial current distribution* x_0 *is said to be null controllable, if there exists a finite time* t_f *and an admissible control voltage law* $u(t)$ *such that the state trajectory* $x(t)$ *satisfies* $x(0) = x_0$, $x(t_f) = 0$. *The set of all null controllable states is called the null controllable region of the system*

Null controllable region - 2

- For the ITER case under investigation, an admissible control voltage law has to satisfy the constraints

$$|u_{ij}(t)| \leq V_{max}\,, \quad i = 1,2,3\,, \quad j = 1,2,\ldots,9\,,$$

  where u_{ij} is the j-th component of the vector u_i, and V_{max} is the maximum power supplies voltage
- By a suitable state-space transformation $\xi = Tx$, the state space equation can be rewritten as

$$\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_s \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \alpha & \omega \\ -\omega & \alpha \end{pmatrix} & 0 \\ 0 & A_s \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_s \end{pmatrix} + \begin{pmatrix} B_u \\ B_s \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

  where the matrix A_s contains the stable modes
- The null controllable region of our system does not change for effect of a state-space transformation
- A procedure to evaluate this null controllable region is based on solving a suitable optimal control problem for the reduced order system, taking into account only the unstable part of our system

Best achievable performance

Best achievable performances for the stabilization of the $n = 1$ resistive wall mode for the selected plasma equilibrium. The first two columns report the maximum values of the voltage and current for the power supplies. The third column reports the maximum recoverable perturbation as measured by the available sensors. Finally the fourth column reports the maximum perturbation as measured by the available sensors during the recovery.

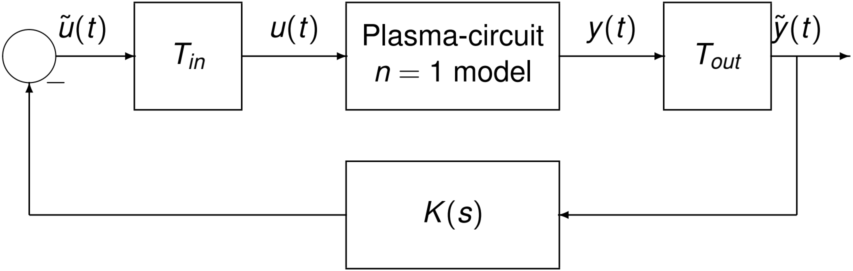
V_{max} [V]	I_{max} [A]	Y_0 [mT]	Y_{max} [mT]
36	16	35.9	46.9
72	16	40.7	43.5
144	16	42.2	43.4
216	16	42.2	43.4

Control requirements

The stabilizing controller is designed with the objective of satisfying the following requirements:

1. it should allow us to obtain a maximum recoverable initial perturbations of the magnetic fields measured by the sensors, as close as possible to the best achievable performance
2. it should be able to recover an initial $n = 1$ perturbation as fast as possible, compatibly with the constraints on the active coil voltages and currents
3. it should avoid to generate a magnetic field containing toroidal harmonics different from $n = 1$

Control scheme



Design - 1

- The matrices T_{in} and T_{out} are defined in such a way to satisfy the third requirement
- The system to be controlled has six control inputs and two measured outputs, and hence $K(s)$ is a multivariable controller
- One possibility is to design it on the basis of the LQG approach. Since the system to be controlled is of a very high order, and since the LQG technique provides a controller having the same order of the plant, a model reduction of system
- To this end, for numerical reason, first the improved Davison methods is used to reduce the order from several thousands to several hundreds, and then the balanced truncation technique is used to reduce the order of the final model to about 40

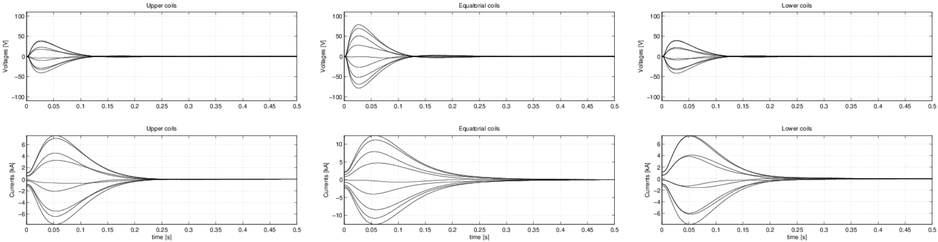
Design - 2

- As it is well known, an LQG controller takes advantage of the separation principle
- Hence, first a state feedback gain is designed with the aim of optimizing a state and control input integral quadratic index, then a Kalman filter is designed in such a way to optimize in a least square sense the estimation of the state variables starting from the available measurements
- The state feedback gain is designed in such a way to enlarge as much as possible the region of asymptotic stability of the closed loop system according to the power supplies constraints
- The Kalman filter is designed taking into account the noise statistics of the magnetic measurements

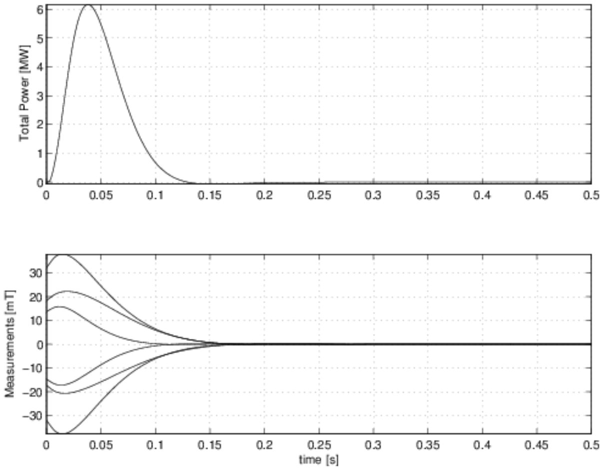
Simulation results

- The controller performance have been validated on the full order system, taking into account all the power supplies constraints.
- The simulation results show that the amplitude of the initial magnetic perturbation is 35 mT which is very close to the maximum recoverable perturbation which is in the order of 40 mT
- The voltages remain always well below their limits: the maximum voltage is reached on the equatorial coils and it is about 80V, 144V being the limit.
- The maximum current, reached again on the equatorial coils is about 12.5 kA, not so far from the limit of 16 kA.
- The perturbation is recovered in less than 0.5 s which is a very small time interval for the ITER time scale.

Voltages and currents



Power and field measurements



Conclusions

- A control scheme has been proposed for the control of RWM instabilities in ITER
- Scope of the proposed control architecture is to stabilize the plant, maximizing the operating region without generating toroidal harmonics different from $n = 1$
- Simulation results, obtained for a suitable configuration of an ITER plasma, show the effectiveness of the proposed approach